

# ESTIMATION OF STABILITY AND CONTROL DERIVATIVES FROM FLIGHT TEST DATA

By

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DEPARTMENT OF AERONAUTICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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# ESTIMATION OF STABILITY AND CONTROL DERIVATIVES FROM FLIGHT TEST DATA

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In partial Fulfilment of the Requirements  
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By  
FLT. LT. B. S. YADAV

to the

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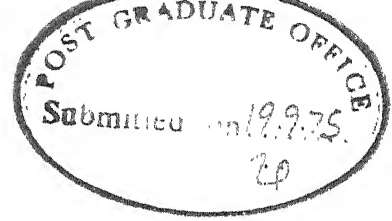
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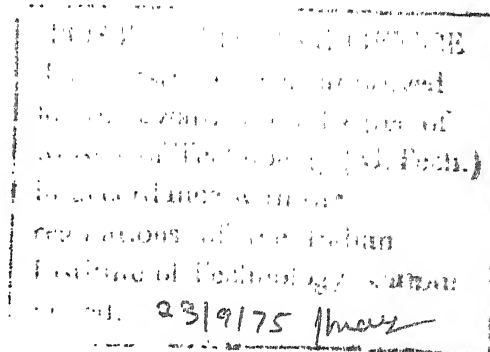
CERTIFICATE

This is to certify that the work " ESTIMATION OF STABILITY AND CONTROL DERIVATIVES FROM FLIGHT TEST DATA" has been carried out under my supervision and has not been submitted elsewhere for a degree.

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## SUMMARY

In the present work, the stability and control derivatives have been determined from the flight test data. For extraction of aircraft parameters measured values of test inputs and the resulting output responses are required. The measured responses are compared with the calculated responses by assuming initial values of the parameters. The square of the difference between the measured and calculated responses is minimized by the modified Newton Raphson method and the Davidon-Fletcher-Powell method. The values of the parameters are updated at each iteration and the process is continued till the minima is achieved. The results obtained for a light subsonic aircraft by the two methods have been compared. It is found that all the important derivatives extracted by the two methods, are in close agreement. The computed time histories by the two methods match well with the flight data.

# LIST OF SYMBOLS

$A$	stability matrix (PXP)
$A_{ij}$	null matrix except for the $i$ - $j^{\text{th}}$ element which equals (PXP)
$A_i$	$i^{\text{th}}$ row of the stability matrix (1XP)
$a_{ij}$	$i$ - $j^{\text{th}}$ element of $A$
$B$	control matrix (PXQ)
$B_{bij}$	null matrix except for the $i$ - $j^{\text{th}}$ element which equals (PXQ)
$B_i$	$i^{\text{th}}$ row of the control matrix (1XQ)
$b_{ij}$	$i$ - $j^{\text{th}}$ element of $B$
$b$	span $m$ (ft)
$G$	augmented $A$ and $B$ matrices (PX(P+Q))
$G_i$	$i^{\text{th}}$ row of the $G$ matrix (1X(P+Q))
$c$	vector of unknown coefficients (mX1)
$c_i$	$i^{\text{th}}$ element of the $c$ vector
$c_{m_\alpha}$	variation of pitching moment coefficient with angle of attack, $\text{rad}^{-1}$
$c_{m_{\delta_e}}$	variation of pitching moment coefficient with elevator angle, $\text{deg}^{-1}$ or $\text{rad}^{-1}$
$c_T$	vector of actual values of unknown coefficients (EX1)
$D_1$	weighting matrix for observation vector (RXR)
$G$	partition of matrix relating the state vector to the observation vector (R-P)XP)
$g$	acceleration due to gravity, $\text{m/sec}^2(\text{ft/sec}^2)$

$g$	vector of observation biases (RX1)
$g_i$	$i^{\text{th}}$ element of $g$
$H$	partition of matrix relating the control vector to the observation vector (R-P)XQ)
$I$	identity matrix
$I_{xx}, I_{yy}, I_{zz}$	moment of inertia about x,y and z axes
$J$	cost functional or weighted mean-square-fit error
$l$	number of time samples
$m$	number of unknowns in $c$ vector
$P$	number of state variables
$p$	roll rate, rad/sec or deg/sec
$Q$	number of control variables
$q$	pitch rate, rad/sec or deg/sec
$R$	number of observation variables
$r$	yaw rate, rad/sec or deg/sec
$T$	total time, sec
$t$	intermediate or incremental time, sec
$u$	control vector (QX1)
$V$	velocity, m/sec (ft/sec)
$x$	state vector (PX1)
$x_i, x_j$	$i^{\text{th}}$ and $j^{\text{th}}$ component of $x$
$Y$	side force divided by mass and velocity, rad/sec
$y$	observation vector (RX1)
$y_i$	$i^{\text{th}}$ element of the $y$ vector

$z$	measurement of state variables (PX1)
$\dot{z}$	measurement related to derivatives of state variables ((R-P)X1)
$z$	measurement of observation vector (RX1)
$\alpha$	angle of attack of X-axis, rad or deg
$\beta$	angle of sideslip, rad or deg
$\Delta$	increment
$\delta$	first variation
$\delta_a$	aileron deflection, rad or deg
$\delta_e$	elevator deflection, rad or deg
$\delta_r$	rudder deflection, rad or deg
$\tau$	auxiliary time variable, sec
$\phi$	bank angle, rad or deg
$\omega$	frequency, rad/sec
$\nabla_c(.)$	gradient of (.) with respect to $c$
$\nabla_g(.)$	gradient of (.) with respect to $g$

Dimensional stability and control derivatives:

$L_p$	Dimensional variation of rolling moment with roll rate, $\text{sec}^{-1}$
$L_r$	Dimensional variation of rolling moment with yaw rate, $\text{sec}^{-1}$
$L_\beta$	Dimensional variation of rolling moment with side slip angle, $\text{sec}^{-2}$
$N_p$	Dimensional variation of yawing moment with roll rate, $\text{sec}^{-1}$
$N_r$	Dimensional variation of yawing moment with yaw rate, $\text{sec}^{-1}$

$N_{\beta}$	Dimensional variation of yawing moment with side slip angle, $\text{sec}^{-2}$
$Y_p$	Dimensional variation of side force with roll rate, $\text{m/sec}^2(\text{ft/sec}^2)$
$Y_{\beta}$	Dimensional variation of side force with side slip angle, $\text{m/sec}^2(\text{ft/sec}^2)$
$L_{\delta_a}$	Dimensional variation of rolling moment with aileron angle, $\text{sec}^{-2}$
$L_{\delta_r}$	Dimensional variation of rolling moment with rudder-angle, $\text{sec}^{-2}$
$N_{\delta_a}$	Dimensional variation of yawing moment with aileron angle, $\text{sec}^{-2}$
$N_{\delta_r}$	Dimensional variation of yawing moment with rudder angle, $\text{sec}^{-2}$

## Subscripts:

$i$	$i^{\text{th}}$ row or component
$j$	$j^{\text{th}}$ column or component
$k$	iteration index
$me$	measured

## Superscripts:

$i, j$	index representing time of sample
$T$	matrix transpose

A dot over a quantity denotes the time derivative of that quantity. Principal axes are used throughout.

LIST OF FIGURES

FIG. NO.

1. Basic concept of aircraft parameter estimation technique.
2. Logical flow diagram for computer program for the modified Newton Raphson and Davidon-Fletcher-Powell techniques.
3. Comparison of the time histories measured in flight and computed by the modified Newton Raphson method.
4. Comparison of the time histories measured in flight and computed by D.F.P. method.

# CHAPTER 1

## INTRODUCTION

### 1.1 GENERAL:

Over the past several years, there had been a renewed interest in determining dynamic aircraft parameters such as stability and control derivatives, from flight test measurements. The need for these data has long persisted. However, only recently highly automated data acquisition systems and advanced estimating techniques have been available, that one can extract such information efficiently.

Various methods of estimating aircraft parameters, i.e. stability and control derivatives, by use of wind tunnel measurements or from theory, are in existence. These derivatives can be used to compute time histories of various motions of the aircraft. It is important to realize that the computed motions and stability are meaningless, unless the equations of motion and stability derivatives are truly representative of the aircraft under consideration. Wind tunnel measurements are usually made with small models, and Reynolds number, roughness, tunnel wall effects, Mach number etc., generally are not properly scaled to simulate the full aircraft. It, therefore, becomes necessary to assess the correctness of the derivatives, so determined by the wind tunnel measurements. One

method is to obtain the derivatives by flight testing of aircraft. In this method from a given group of measurements of velocities, positions and accelerations taken over a time interval, we have to determine whether the stability and control derivatives can be estimated. The process of extracting numerical values for the aerodynamic stability and control derivatives, is known as aircraft parameter identification.

The following are the needs for identification:-

To provide input information to aircraft simulators.

To provide basis for design of flight control systems.

The stability and control derivatives define a given aircraft more uniquely than the response mode criteria (As stated in various flying qualities specifications like MIL-F-85). It is thus anticipated that in future these parameters will ultimately play a major role in the design, testing and certification of aircraft.

### 3 BACK-GROUND AND BRIEF HISTORICAL DEVELOPMENT:

One of the first flight test programmes to obtain quantitative measurements of aircraft aerodynamic characteristics was reported by Norton in 1919. Lift and drag coefficients are determined by equating the lift to weight and drag to engine thrust.

Soule and Wheatley (e.g.1) appear to be the first to have determined all the major longitudinal stability and control derivatives of an aircraft from flight test data. This analysis used simplified equations representing one-degree-of freedom of motion. Equations were solved for one parameter at a time, assuming values for other parameters based on wind-tunnel tests. This basic approach was used till middle of 1940's.

Milliken (1947) pointed out that the increasing use of automatic control systems required more accurate modelling of the aircraft dynamic characteristics. These factors coupled with the research engineer's motivation to improve the accuracy of flight results, stimulated the development of several new techniques for determining stability and control derivatives from flight data.

In the late 1940's through mid 1950's servomechanism theory was expanded rapidly and the frequency-domain techniques of Nyquist and Bode (see e.g.2,3) were popular. A disadvantage of this approach was the considerable flight time required to sweep through all the frequencies of interest at each flight condition.

The problem of determining stability and control derivatives is based on a linearized, small-perturbation model of the aircraft dynamics. Hence it was natural to consider using

a linear least-square fit of flight data to the linearized equations of motion as was done by Greenberg(4) in 1951. In 1954 Shinbrot (5) developed a generalized least-square method which encompassed the earlier least-square methods. During the period when Greenberg and Shinbrot developed their methods, the digital computers were not available. These two methods required extensive calculations to match the computed response with the flight measured response of the aircraft. Thus a large volume of flight data had to be processed manually for extracting the stability and control derivatives.

'Analog Matching' (see e.g. 6,7) technique was used as early as 1951 to check aircraft parameters determined by other methods. Even though the technique of analog matching has greatly improved over the period, the accuracy of the results is highly dependent on the skill of an individual operator. If several parameters are to be determined, this technique requires excessive number of man-hours to obtain an acceptable solution.

Although several attempts were made throughout the 1950's and early 1960's to improve techniques, the effort was relatively small. Two factors caused a revolution in aircraft parameter estimation techniques starting in mid-1960's. These were:

1. Highly automated data acquisition systems were becoming standard in flight testing.

2. Large-capacity, high-speed digital computers were available to solve complicated algorithms efficiently. The ability to transfer the flight data directly to the computer with no manual operations on the data, helped in increasing the use of flight-testing techniques for evaluating aircraft parameters.

The interest in parameter estimation was renewed in 1968. Larson (8) applied the method of quasi-linearization at The Cornell Aeronautical Laboratory. Taylor and Iliff (9) used basically the same approach, but referred to it as the modified Newton-Raphson technique, at the NASA Flight Research Centre. Denery (10) applied the general theory of system identification to air vehicles for extracting aircraft parameters. Mehra (11) used the method of stochastic approximations to solve system identification problems. Later on the same approach was applied for aircraft identification (see 13).

#### 1.4 AIRCRAFT PARAMETER ESTIMATION:

The general problem of parameter estimation is to determine certain characteristics of the physical system from experimental test data. Measurements are made of the test inputs and the resulting output responses that depend in some way on the system characteristics to be determined. Parameter estimation technique is the process of estimating characteristic from the input/output measurements.

The general concept of the aircraft parameter estimation technique is illustrated in Fig. 1. The various aspects of the problem are:

- (i) Mathematical Model
- (ii) Estimation Criterion
- (iii) Computational Algorithm
- (iv) Instrumentation and Data Acquisition
- (v) Test Inputs

#### 1.4.1 Mathematical Model:

A model must be selected that adequately represents the aircraft characteristics to be measured. For the present problem the aircraft is assumed to be rigid and the linearized equations of motion are considered. However, in other instances a more complex model may be necessary, such as at very high angles of attack a non-linear model may be required. An inappropriate model can degrade the accuracy of the parameter estimates.

#### 1.4.2 Estimation Criterion:

There must be some means of assessing fit of the computed response to the measured response. To implement this 'criterion function' is used. It is usually some form of the integral square of the error between computed and measured response. The best estimate of the parameters is the set of parameters that minimizes the criterion function.

#### 1.4.3 Computational Algorithm:

The criterion function is often non-linear with respect to the parameters to be estimated; therefore it has to be minimized by an iterative computational algorithm. Important factors in selecting the minimization algorithm are convergence, computation efficiency, local minima etc.

#### 1.4.4. Instrumentation and Data Acquisition System:

Aircraft parameter estimation is highly dependent on the quality of the flight measured data. In reality bias and random errors can arise from improper location or orientation of sensors, calibration of measurement and recording system. Other errors can be introduced from electrical noise, engine vibrations, inappropriate signal filters, air turbulence etc. Any elimination of errors, noise or uncertainties within the data acquisition system will improve the accuracy of the estimates. A comprehensive discussion of flight-test instrumentation for aircraft parameter estimation is available in Ref. 12.

#### 1.4.5 Test Input:

As a minimum requirement, the test input must excite the principal response modes that depend on the parameters to be determined. In the example considered in this work, a combination of rudder and aileron pulses adequately excited the

lateral-direction motion. However, a question arises whether one type of control input might be better than the other, in the sense that it provides better estimates. Several papers have considered this question (13) but the concept has not been fully explored in a flight-test application (14).

#### 1.4 PRESENT WORK:

In the present work, we are interested in analysing the flight data, in order to obtain the stability and control derivatives. The details of flight data acquisition and those of test inputs have not been considered here. A subsonic light aircraft is selected for the analysis in the lateral-directional mode, ~~as~~ the flight data for the same could be obtained from Ref. 17.

The process of extraction of stability and control derivatives involves the measurement of test inputs and the resulting output responses. The measured responses are compared with the calculated responses by assuming initial values of the parameters. The square of the difference between the measured and calculated responses is minimized by the modified Newton Raphson method and the Davidon-Fletcher-Powell method. The values of the parameters are updated at each iteration and the process is continued till the minima is achieved.

There is no direct reference in the literature about the use of the Davidon-Fletcher-Powell method for the flight dynamics use and its results. It was, therefore, decided to use this method to find out if it could be applied for solving aircraft dynamics problems.

The details of the methods for determination of stability and control derivatives from flight data are presented in Chapter 2. The identification problem is stated and a brief review of the various methods used for aircraft parameter identification has been carried out. The modified Newton Raphson method and the Davidon-Fletcher-Powell minimization techniques are discussed in detail.

The computational details, results, discussions and conclusions are given in Chapter 3. The parameters estimated by the two methods and the time histories are compared. Finally the source of errors are pointed out and the suggestions are made for further work.

## CHAPTER 2

### METHODS FOR DETERMINATION OF STABILITY AND CONTROL DERIVATIVES FROM FLIGHT DATA

#### 2.1 INTRODUCTION:

A general statement of the problem of aircraft parameter identification, as applied to the extraction of lateral-directional stability and control derivatives, is presented here. A brief review of the previous methods is presented first. Subsequently, the two techniques chosen for the present study viz. the modified Newton Raphson technique and the Davidon-Fletcher-Powell minimization techniques are described in details.

#### 2.2 STABILITY DERIVATIVE DETERMINATION: STATEMENT OF THE PROBLEM

For most of the aircraft dynamics problems, the longitudinal mode is usually separated from the lateral-direction mode, because the resulting error is small. The estimation of stability and control derivatives in lateral-direction mode is more complex than the longitudinal mode. This work deals with the problem of stability and control derivative estimation in the lateral-direction mode.

The mathematical model often used to describe lateral-

directional

Airplane dynamics can be expressed as a system of linear ordinary differential equations with constant-coefficients, in the following form (see Appendix A)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) \quad (2.1)$$

where

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} \quad \mathbf{x}(t) = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} \quad u(t) = \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix}$$

The last column of matrix B represents the effect of uncertain bias on the measurement of  $\dot{p}$ ,  $\dot{r}$  and  $\dot{\beta}$ .

Let  $z(t)$  and  $\hat{z}(t)$  be the noise contaminated measurements of  $\mathbf{x}(t)$  and  $\dot{\mathbf{x}}(t)$  respectively. The control inputs  $u(t)$  are considered to be noise-free.

Given  $z$ ,  $\dot{z}$  and  $u$ , the problem is to determine certain unknown elements of matrices A and B, i.e. the stability and control derivatives.

### 2.3 PREVIOUS METHODS : BRIEF REVIEW:

Some of the methods used earlier for determining stability and control derivatives are described below:

#### 2.3.1. Simplified Equations Method:

For selected type of responses, the effect of only a few coefficients dominates, thus permitting the use of simple expressions to determine these coefficients. Wolowicz (12) obtained good approximations for some of the longitudinal stability derivatives by keeping only the dominant terms when equations of motion have been solved for a particular derivative. For example,  $C_{m\delta_e}$  can be determined from the initial portion of rapid pulse maneuver by :

$$C_{m\delta_e} \approx \frac{I_{yy}}{\frac{1}{2} \rho V^2 S c} \frac{\Delta \dot{q}}{\Delta \delta_e} \quad (2.2)$$

Similarly,  $C_{m\alpha}$  can be approximated to within 3% accuracy from the relation,

$$C_{m\alpha} \approx \frac{I_{yy}}{\frac{1}{2} \rho V^2 S c} W_{n.s.p.}^2 \quad (2.3)$$

where  $W_{n_{s.p}}$  is the frequency of short period oscillations

Wolowicz (12) also gave some simplified approximations for the lateral stability derivatives. However, because of the more complex behaviour of the airplane and large number of derivatives involved, the lateral-directional control and stability derivatives are not as readily and reliably determined by the use of approximate equations, as are the longitudinal derivatives.

The disadvantages of this method are:

- 1, Only some of the primary unknown coefficients of stability and control derivatives can be determined,
- 2, The forms of response that can be analyzed are very restrictive i.e., effect of control must either be dominant or negligible,

### 2.3.2. Analog-Matching Method:

In analog matching technique, the values of stability and control derivatives are assumed and motions are computed for the same control inputs as those used in flight tests. Comparisons are made between the computed and flight-measured responses. If any of the responses do not compare favourably, some of the derivatives are changed and the motions are computed again. The process is repeated until satisfactory

agreement is obtained between the estimated and flight-measured responses. The ability to converge to a set of acceptable derivatives, or of even finding acceptable agreement between computed and flight motions, is largely a matter of experience.

The main disadvantages of this technique are:

1. Analog-matching depends quite heavily on the experience of the operator. However, this method can be used as a back-up method and has given satisfactory results in many cases.
2. The method works successfully only when a single control surface is moved at a time so that the maneuvers are simple. (see 12).
3. When the maneuvers are made with stability augmentation system, the data is difficult to analyze.
4. This method is extremely time consuming.

### 2.3.3 Least-Squares Method:

Least-squares method assumes a performance criterion which minimizes the square of the state equation error, by substituting the measured value of the state and its derivatives i.e. it minimises the following cost functional:

$$J = \int_0^T (\dot{z} - Az - Bu)^T (\dot{z} - Az - Bu) dt \quad (2.4)$$

To derive expressions for the values of  $A$  and  $B$  that minimizes Eq. (2.4), let

$$C = \begin{bmatrix} A \\ B \end{bmatrix}$$

Then,

$$J = \int_0^T \dot{z}^T \dot{z} dt - 2 \int_0^T \dot{z}^T c \begin{bmatrix} -\frac{z}{u} - \end{bmatrix} dt + \int_0^T \begin{bmatrix} -\frac{z}{u} - \end{bmatrix} c^T c \begin{bmatrix} -\frac{z}{u} - \end{bmatrix} dt \quad (2.5)$$

The minimization is achieved by taking the first derivative of Eq. (2.5) w.r.t.  $c$  i.e.

$$\frac{\partial J}{\partial c} = 0$$

This yields,

$$c^T = \left\{ \int_0^T \begin{bmatrix} -\frac{z}{u} - \end{bmatrix} \begin{bmatrix} -\frac{z}{u} - \end{bmatrix}^T dt \right\}^{-1} \int_0^T \begin{bmatrix} -\frac{z}{u} - \end{bmatrix} \dot{z}^T dt \quad (2.6)$$

Eq. (2.6) is the desired solution by the method of least-squares. It has the advantage of compact form but disguises the independence of each of the equations to be minimized. This can be shown in the following manner. Consider only the first state equation from Eq. (2.1),

$$\dot{x}_1(t) = A_1 x(t) + B_1 u(t)$$

where

$$A_1 = \begin{bmatrix} L_p & L_r & L_\beta & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \end{bmatrix}$$

$$C_1 = \begin{bmatrix} A_1^T & B_1^T \end{bmatrix}$$

Then

$$\begin{aligned} J &= \int_0^T \left[ \dot{p}_{m_e} - L_p p_{m_e} - L_r r_{m_e} - L_\beta \beta_{m_e} - L_{\delta_a} \delta_a \right. \\ &\quad \left. - L_{\delta_r} \delta_r - L_o \right]^2 dt \\ &= \int_0^T \left\{ \dot{p}_{m_e} - c_1 \begin{bmatrix} -\frac{z}{u} & -1 \end{bmatrix} \right\}^2 dt \end{aligned}$$

where the subscript  $m_e$  denotes the measured value.

Once again taking the first derivative and setting the resulting expression to zero, gives

$$c_1^T = \left\{ \int_0^T \begin{bmatrix} -\frac{z}{u} & -1 \end{bmatrix} \begin{bmatrix} -\frac{z}{u} & -1 \end{bmatrix}^T dt \right\}^{-1} \int_0^T \dot{p}_{m_e} \begin{bmatrix} -\frac{z}{u} & -1 \end{bmatrix} dt \quad (2.7)$$

Now  $c_1$  is the first row of the matrix  $c$  and  $\dot{p}_{m_e}$  is the first element of  $\dot{z}$ , which makes it apparent that the elements of the first row of the  $c$  matrix are independent of all the elements of  $\dot{z}$  except the first,  $\dot{p}_{m_e}$ . A similar relationship

can be easily shown for the other rows of c matrix.

The disadvantages of this method are:

1. The row independence (shown above by Eq. 2.7) is one of the drawbacks of this method, in that only one of the measured state variables is used in determining a given row of c matrix. If one of the signals has not been measured, the least-squares method does not provide an estimate of the derivatives related to that signal.

This independence also illustrates that the estimate of one row of the c matrix is obtained independently of the other rows, and no 'trade-off' can be made between elements in different rows to improve the match.

2. If two or more of the measured responses are linearly related, this method gives an absurd solution or no solution (15).

3. This method gives excessive variance of the estimated coefficients (16).

## 2.4 MODIFIED NEWTON-RAPHSON AND DAVIDON-FLETCHER-POWELL TECHNIQUES:

### 2.4.1 General:

These techniques are means of selecting those parameter values which best fit an assumed model to a data set

according to a particular error criterion. The error criterion is more general than the least-squares criterion in that it permits the fit error to  $p$ ,  $r$ ,  $\beta$  and  $\phi$  to be minimised as well as the fit error to  $\dot{p}$ ,  $\dot{r}$  and  $\dot{\beta}$ . These techniques also enable one to use a priori values of the stability derivatives, bias terms, and initial conditions to improve the fit of the equations to <sup>match</sup> flight test data.

The mathematical model chosen here describes the lateral-directional motions of an aircraft by linear, constant coefficient differential equations. Consider the following model (see Appendix A)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (2.8)$$

$$\text{and } \mathbf{y} = \begin{bmatrix} -\frac{\mathbf{I}}{\mathbf{G}} & -\end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{0} \\ \mathbf{H} \end{bmatrix} u \quad (2.9)$$

where,

The vector  $\mathbf{y}$  is the set of out-put response quantities:

$$\mathbf{y} = [p \ r \ \beta \ \phi \ \dot{p} \ \dot{r} \ \dot{\beta}]^T$$

The matrices  $\mathbf{A}$  and  $\mathbf{B}$  are defined in Eq. (2.1)

$$[\mathbf{I}] = \text{Identity Matrix}$$

$$[\mathbf{0}] = \text{Null matrix}$$

$$G = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \end{bmatrix}$$

$$H = \begin{bmatrix} L_{\delta a} & L_{\delta r} & L_o \\ N_{\delta a} & N_{\delta r} & N_o \\ Y_{\delta a} & Y_{\delta r} & Y_o \end{bmatrix}$$

Let  $z$  denote the measurement of the actual aircraft response quantities corresponding to the computed output response quantities,  $y$ . The measured response  $z$  would not be exactly the same as  $y$  because of,

- (i) The measurement errors
- (ii) The difference between the actual and the assumed linear mathematical model, Eq. (2.9).

The objective is to minimize the difference between  $z$  and

The appropriate cost functional is,

$$J = \int_0^T (z-y)^T D_1 (z-y) dt \quad (2.10)$$

Where  $D_1$  is a weighting matrix reflecting the relative confidence in the measurements.

After specifying the cost functional  $J$ , an algorithm is to be chosen for minimizing this cost functional. Many methods are available for non-linear minimization. However, for this particular work a modified Newton-Raphson technique due to Niff and Taylor (9) and the Davidon Fletcher Powell technique are selected. They are described below.

#### 2.4.2 Modified Newton-Raphson Minimization Technique

For convenience, let us define a column vector  $c$  of the unknowns to be estimated. The elements of  $c$  are some or all of the unknown elements of  $A$  and  $B$  of the noise biases  $g$  and of the initial conditions  $x_i(0)$ , e.g.

$$c = [a_{ij}, b_{ij}, g_i, x_i(0)]$$

In all subsequent calculations, the noise biases  $g_i$  and the variable initial condition  $x_i(0)$  have been neglected.

The Newton-Raphson technique is an iterative method for finding a zero of a nonlinear function of several parameters, or in this instance, a zero of the gradient of the cost functional  $J$ , i.e.

$$\nabla_c J = 0$$

Consider a two-term Taylor's series expansion of  $\nabla_c J$  about the  $k^{\text{th}}$  value of  $c_k$ :

$$(\nabla_c J)_{k+1} \approx (\nabla_c J)_k + (\nabla_c^2 J)_k \Delta c_{k+1} \quad (2.11)$$

where:

$$\Delta c_{k+1} = (c_{k+1} - c_k)$$

and  $(\nabla_c^2 J)_k$  is the second gradient of the cost functional w.r.t.  $c$ , at the  $k^{\text{th}}$  iteration. If equation (2.11) is a sufficiently close approximation, the change in  $c$  on the  $(k+1)^{\text{th}}$  iteration to make  $(\nabla_c J)_{k+1}$  approximately zero is,

$$\Delta c_{k+1} = - [(\nabla_c^2 J)_k]^{-1} (\nabla_c J)_k \quad (2.12)$$

This is Newton-Raphson algorithm.

To evaluate the change in parameter value  $\Delta c_{k+1}$  by Eq. (2.12), the values of  $(\nabla_c J)_k$  and  $(\nabla_c^2 J)_k$  are needed. The gradient of  $J$  with respect to the vector  $c$  can be expressed in terms of the gradient of  $(z-y)$  by differentiating Eq. (2.10) as,

$$\nabla_c J = 2 \left\{ \int_0^T (z-y)^T D_1 \nabla_c (z-y) dt \right\}^T \quad (2.13)$$

In order to evaluate  $\nabla_c J$  the term  $\nabla_c(z-y)$  is required to be specified in addition to the terms already defined. This can be obtained by differentiating Eq. (2.9),

as follows:

$$\begin{aligned} \nabla_c (z-y) = & - \left\{ \nabla_c \left[ -\frac{I}{G} - \right] \right\} x - \left[ -\frac{I}{G} - \right] \nabla_c x - \left\{ \nabla_c \left[ -\frac{0}{H} - \right] \right\} u \\ & - \left[ -\frac{0}{H} - \right] \nabla_c u \end{aligned} \quad (2.14)$$

The quantity  $\nabla_c (z-y)$  can be expressed in terms of various partial derivatives. These partial derivatives with respect to the individual coefficients of  $c$  ( $a_{ij}$ ,  $b_{ij}$ ) are,

$$\frac{\partial \left[ -\frac{I}{G} - \right]}{\partial a_{ij}} x = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ x_j \\ 0 \\ 0 \end{bmatrix} \quad \frac{\partial \left[ -\frac{0}{H} - \right]}{\partial a_{ij}} u = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}$$

$$\frac{\partial \left[ -\frac{I}{G} - \right]}{\partial b_{ij}} x = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} \quad \frac{\partial \left[ -\frac{0}{H} - \right]}{\partial b_{ij}} u = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ u_j \\ \vdots \\ 0 \end{bmatrix}$$

where  $x_j$  and  $u_j$  appear in  $i+j$  row of the vector,

The gradient of  $x$  with reference to  $c$  is given by

$$\nabla_c x = \begin{bmatrix} \frac{\partial p}{\partial c_1} & \frac{\partial p}{\partial c_2} & \dots & \frac{\partial p}{\partial c_m} \\ \frac{\partial r}{\partial c_1} & \frac{\partial r}{\partial c_2} & \dots & \frac{\partial r}{\partial c_m} \\ \frac{\partial \beta}{\partial c_1} & \frac{\partial \beta}{\partial c_2} & \dots & \frac{\partial \beta}{\partial c_m} \\ \frac{\partial \phi}{\partial c_1} & \frac{\partial \phi}{\partial c_2} & \dots & \frac{\partial \phi}{\partial c_m} \end{bmatrix}$$

The elements of gradient  $x$  can be determined in the following manner. Expanding  $x$  at  $(k+1)^{th}$  iteration by Taylor series, about  $x^k$  and retaining terms upto first derivative

$$x^{k+1} = x^k + \dot{x}^k \Delta$$

Substituting the value of  $\dot{x}$  from Eq. (2.1), we get,

$$x^{k+1} = x^k + \Delta [Ax^k + Bu^k] \quad (2.15)$$

Taking partial derivative of Eq. (2.15) with respect to  $a_{ij}$  and  $b_{ij}$ , it becomes,

$$\begin{aligned} \frac{\partial x^{k+1}}{\partial a_{ij}} &= \frac{\partial x^k}{\partial a_{ij}} + \Delta \left[ A \frac{\partial x^k}{\partial a_{ij}} + \frac{\partial A}{\partial a_{ij}} x^k \right] \\ \frac{\partial x^{k+1}}{\partial b_{ij}} &= \frac{\partial x^k}{\partial b_{ij}} + \Delta \left[ A \frac{\partial x^k}{\partial b_{ij}} + \frac{\partial B}{\partial b_{ij}} u^k \right] \end{aligned} \quad (2.16)$$

The initial conditions are invariant with respect to  $a_{ij}$  and  $b_{ij}$ , i.e.

$$\frac{\partial x(0)}{\partial a_{ij}} = 0 \quad \text{and} \quad \frac{\partial x(0)}{\partial b_{ij}} = 0$$

The elements of the gradient  $x$  can now be determined by Eq. (2.16). An alternate procedure to determine the elements of gradient  $x$ , due to Taylor (9) is given in Appendix B.

Thus all the terms in Eq. (2.14) have been evaluated;  $\nabla_c J$  can now be evaluated by Eq. (2.13).

The difference between the measured and computed responses  $(z-y)$ , can be represented as quasi-linear with respect to a change in the unknown coefficients, i.e.

$$[z-y]_k \approx [z-y]_{k-1} + \nabla_c [z-y]_k \Delta c_k$$

Using this approximation in the cost functional results in the following first and second gradients:

$$\nabla_c J = 2 \left\{ \int_0^T [(z-y)_k]^T D_1 [\nabla_c (z-y)]_k dt \right\}^T \quad (2.17)$$

$$\begin{aligned} \nabla_c^2 J = 2 \int_0^T & \left[ \left\{ \nabla_c (z-y) \right\}_k^T D_1 [\nabla_c (z-y)]_k dt + \right. \\ & \left. 2 \int_0^T \left\{ \nabla_c^2 (z-y) \right\}_k^T D_1 (z-y)_k dt \right] \quad (2.18) \end{aligned}$$

The second term of  $\nabla_c^2 J$  diminishes, as the response error  $(z - y_k)$  diminishes. The modified Newton-Raphson technique as

developed by Taylor (9) neglects this term. Thus, the final relation for  $\nabla_c^2 J$  becomes

$$\nabla_c^2 J = 2 \int_0^T \left\{ \left[ \nabla_c (z-y) \right]_k \right\}^T D_1 \left[ \nabla_c (z-y) \right]_k dt \quad (2.19)$$

Now, substituting the values of  $\nabla_c J$  and  $\nabla_c^2 J$  in Eq. (2.12), the modified Newton-Raphson algorithm becomes

$$\Delta c_{k+1} = - \left\{ \int_0^T \left\{ \left[ \nabla_c (z-y) \right]_k \right\}^T D_1 \left[ \nabla_c (z-y) \right]_k dt \right\}^{-1} * \\ \int_0^T \left\{ \left[ \nabla_c (z-y) \right]_k \right\}^T D_1 (z-y)_k dt \quad (2.20)$$

All the terms in Eq. (2.20) involve only the first gradient of  $(z-y)$ . This greatly reduces the computation time and the approximation improves as the solution is approached.

For computational purposes, the integrals are treated as summations. In the indicial notations the Eq. (2.20) then becomes.

$$\Delta c = - \left\{ \sum_{i=1}^1 \left[ \nabla_c (z^i - y^i) \right]^T D_1 \nabla_c (z^i - y^i) \right\}^{-1} * \\ \sum_{j=1}^1 \left[ \nabla_c (z^j - y^j) \right]^T D_1 (z^j - y^j) \quad (2.21)$$

where subscripts  $i$  and  $j$  are the indices indicating the time sample, and  $l$  is the total number of samples.

Equation (2.21) is the final form of the modified Newton-Raphson algorithm:

#### 2.4.3. Davidon-Fletcher-Powell (D.F.P.) Minimization Technique:

In the D.F.P. technique the local Hessian matrix  $(\nabla_c^2 J)_k$  is replaced by an approximate matrix  $H_{i,j}^k$ . The method of computing this matrix completely eliminates the need for evaluating second derivatives and performing matrix inversions, and yet the sequence of iterations converges quadratically to the minimum point (18).

After selecting a starting point, a direction of search is computed as follows (19):

$$M_1^k = \frac{-\sum_{j=1}^m H_{1,j} \left(\frac{\partial J}{\partial c_j}\right)}{\left[\sum_{l=1}^m \left\{ \sum_{j=1}^m H_{l,j} \left(\frac{\partial J}{\partial c_j}\right) \right\}^2\right]^{1/2}} \quad (2.22)$$

where  $i, j, l = 1, 2, \dots, m$ ,  $m$  being the number of unknowns  $c_i$ , and  $k$  is the iteration index.  $M_1$  are the direction vector components,  $\frac{\partial J}{\partial c_j}$  are the gradient vector components and  $H_{i,j}$  are the elements of a positive definite matrix ( $m \times m$ ), which is initially chosen to be identity matrix. It is evident from Eq. (2.22) that the initial direction of the search is

the path of steepest descent.

A one-dimensional search is conducted in the direction chosen by Eq. (2.22) until a minimum is located utilizing the relation,

$$c_i^{k+1} = c_i^k + \alpha^* M_i^k \quad (2.23)$$

where  $\alpha^*$  is the step size in the direction of search.

Now a convergence check is made. If the convergence is achieved, the process is terminated, otherwise, the matrix  $H^{k+1}$  is calculated as follows:

$$H^{k+1} = H^k + M^k - L^k$$

where

$$M^k = \frac{\Delta c^k (\Delta c^k)^T}{(\Delta c^k)^T ((\Delta G)^k)}$$

$$L^k = \frac{H^k (\Delta G)^k [(\Delta G)^k]^T H^k}{|(\Delta G)^k|^T H^k (\Delta G)^k} \quad (2.24)$$

$$\Delta c^k = c^{k+1} - c^k$$

$$(\Delta G)^k = \left( \frac{\partial J}{\partial c} \right)^{k+1} - \left( \frac{\partial J}{\partial c} \right)^k$$

The value of updated matrix  $H$  is substituted in Eq. (2.22) and a new one-dimensional search is made. The

process is repeated till the convergence is achieved.

Eq. (2.22) to (2.24) is the D.F.P. algorithm (see 19).

## CHAPTER 3

### COMPUTATIONAL DETAILS, RESULTS AND DISCUSSION

#### 3.1 INTRODUCTION:

The flight test data for the lateral-directional mode of a light subsonic aircraft, flying at a speed of approximately 280 M.P.H., are obtained from Ref. 17. This data is used for extraction of lateral-directional stability and control derivatives by the modified Newton Raphson method and the Davidon-Fletcher-Powell method. The computational details are discussed first. The stability and control derivatives obtained by the two methods are compared. The aircraft time histories calculated by using the extracted derivatives are compared with the flight test data. The results are then discussed and the sources of errors are listed. Finally the conclusions are drawn and some suggestions are made for the further work.

#### 3.2 COMPUTATIONAL DETAILS:

The flow diagram depicting the steps involved in the extraction of lateral-directional stability and control derivatives from flight data, is given in Figure 2. The listing of the programmes developed for the modified Newton Raphson method and for the D.F.P. method are given in Appendix C and D respectively. The different variables used for the computations are defined in the program listing.

The input to the programmes consist of the initial guesses of the stability and control derivatives, the control input state, and the aircraft measured responses  $p$ ,  $r$ ,  $\beta$ ,  $\phi$ ,  $\dot{p}$ ,  $\dot{r}$  and  $\dot{\beta}$ . In absence of the specific information and for convenience, the weighting matrix  $D_1$  is taken as identity matrix. Since the values of  $y_{\delta_a}$  and  $y_{\delta_r}$  are generally small, they are taken as zero (see e.g. 16, 21). The value of  $y_{\phi}$  is calculated from the speed of the aircraft. This reduces the number of unknowns to 15 i.e. 8 in matrix A and 7 in matrix B. The responses are measured from 0 to 10 seconds at interval of 0.1 seconds. Thus there are 101 data points of aircraft responses. It is assumed that only the values of  $p$ ,  $r$ ,  $\beta$  and  $\phi$  are known. The values of  $\dot{p}$ ,  $\dot{r}$ , and  $\dot{\beta}$  are calculated by differentiating  $p$ ,  $r$  and  $\beta$  by finite difference method. The time histories input is given in Appendix E.

The Newton Raphson programme consists of the main programme and subroutines CALDIF and MATINV. The subroutine CALDIF calculates the response of the aircraft and  $\nabla_c (z-y)$ , denoted by CDX and H in the programme. The subroutine MATINV calculates the inverse of  $\nabla_c^2 J$ . The increment in the derivative values is calculated in the main program and the values are updated at each iteration. The process is repeated till the minimum value of the cost functional J is achieved. The final values of aircraft responses are given as output in Appendix F.

The D.F.P. programme consists of the main programme and subroutines CALDIF, FUNCT and DEF. The subroutine CALDIF calculates the response of the aircraft and  $\nabla_c (z-y)$ . The subroutine FUNCT calculates the value of cost functional  $J$  and its gradient  $\nabla_c J$ . The subroutine DEF first carries out a one dimensional search until a minima is located. If the convergence is achieved, the process is terminated, otherwise the value of the matrix  $H$  is updated and a new one dimensional search is made. The values of the parameters are updated at each iteration. The iterative process is continued till the minima is achieved. The final values of the aircraft responses are given as output in Appendix G.

### 3.3 RESULTS AND DISCUSSION:

The aircraft time histories calculated from the stability and control derivatives, extracted from the flight data, by the modified Newton Raphson method, are shown in Fig.3. The time histories obtained by the D.F.P. method are shown in Fig.4. It can be seen from these figures that in both the cases the aircraft time histories match the flight data well. The fit obtained by the D.F.P. method is slightly better than that obtained by the modified Newton Raphson method. It can also be noted that none of the responses have shown any divergence from the flight data. There is a slight variation at the peak values of some of the responses. The possible reasons for the same are enumerated in Section 3.4.

The values of the stability and control derivatives obtained by the two methods, are listed in Table 1. It is observed that the values of eight of the twelve derivatives extracted from the flight data by the two methods, are close to each other. The values of the 'damping-in-roll' derivative,  $L_p$ ; the side force derivative due to side-slip,  $Y_\beta$ , and the yaw due to roll derivative,  $N_r$  are sufficiently in error. The sign of the roll due to yaw derivative,  $L_r$  is reversed. One of the reasons of this could be that these parameters are weak parameters, that cannot be accurately identified from flight test data (13).

It has been shown by Frederick (20) that stability derivatives  $L_\beta$ ,  $N_r$ ,  $N_\beta$  are most influential in determining lateral-directional stability. The table 1 shows that these derivatives have been recovered accurately by both the methods. It is also noted that even though some of the unimportant derivatives such as  $N_p$ ,  $Y_\beta$ ,  $L_r$  are sufficiently in error, the time histories are closely matched with the flight data in both the cases.

The modified Newton Raphson technique when used with zero initial values as starting values of the stability and control derivatives, gave large fit errors. An erroneous set of stability and control derivatives was obtained and the calculated responses differed considerably from the measured aircraft dynamics. It is, therefore, essential to provide

appropriate initial approximations to these derivatives which could be used as starting values in the matrices A and B.

The D.F.P. method was also found to be sensitive to the initial starting values of the stability and control derivatives.

The modified Newton Raphson technique first converged to a minimum value of the fit error and then diverged. However, the D.F.P. method never showed this tendency. The D.F.P. method always converged to the local minima. Thus the D.F.P. method can be satisfactorily used in cases where the modified Newton Raphson technique fails to converge. The convergence in case of the modified Newton Raphson method was faster than the D.F.P. method, resulting in less computational time.

Calculations for the problem considered are performed on IBM 7044 computer system. The order of magnitude of the computer time required by the two techniques for solving the present problem, is as follows:

Modified Newton Raphson technique 10-12 minutes

Davidon-Fletcher-Powell technique 35-40 minutes

### 3.4 SOURCE OF ERRORS:

The aircraft behaviour cannot be predicted very accurately by the linear dynamical model of the aircraft.

Similarly, the assumption of no coupling between the longitudinal and the lateral-directional modes, is not accurate enough for the flight conditions of the analysed data. Besides, this the factors that account for the difference in the computed and measured time histories of the aircraft and in the evaluation of stability and control derivatives, are as follows:

1. The input pulse amplitude is not kept small resulting in the violation of assumptions of small perturbations. Increasing amplitude will introduce inertial or aerodynamic nonlinearities.
2. Only the partial data was available for the analysis. The value of the response variables  $\dot{p}$ ,  $\dot{r}$  and  $\dot{\beta}$  were calculated by differentiating  $p$ ,  $r$  and  $\beta$  by the finite difference method. This introduces error in the estimation of  $\dot{p}$ ,  $\dot{r}$  and  $\dot{\beta}$  and thus in the estimation of parameters and the aircraft time histories.
3. Since the results of all parameter identification procedures depend heavily on the quality of the test data available, there is a need for minimizing the instrumentation errors. Some of the more common instrument induced errors include random noise, calibration errors, mounting in accuracies, instrument bias and time lag. Consequently, to achieve reliable results, the data must be conditioned by compensating for instrument shortcomings.

4. The inputs used for exciting the specific modes of an aircraft should be compatible with the derivatives to be extracted. For example, if one is interested in measuring  $C_{L\delta_a}$ , he should perform a maneuver in which  $C_{L\delta_a}$  is a dominant factor, such as rapid roll.

5. The large variations in some of the parameters is due to the fact that these parameters are weak parameters and they cannot be accurately identified from flight test data (13). Though the values of parameters like  $N_p$ ,  $Y_\beta$ ,  $L_r$  vary sufficiently by the two methods but their effect on the time histories is not predominant.

### 3.5 CONCLUSIONS:

Both the techniques viz. the modified Newton Raphson method and the D.F.P. method give good results for the example considered for a light subsonic aircraft. Though the values of unimportant parameters like  $N_p$ ,  $L_r$ ,  $Y_\beta$  vary sufficiently, their effect on the time histories is not predominant. The calculated time histories of the aircraft match well with the flight data for both the methods.

Both the methods are found to be sensitive to the initial values of the derivatives.

Though the modified Newton Raphson method converges faster than the D.F.P. method, it shows a divergence after the

minima is achieved. However, the D.F.P. method never shows such behaviour. It always converges steadily to the minimum value. Thus the D.F.P. method could prove superior in cases where the Newton Raphson method does not show convergence.

### 3.6 SUGGESTIONS FOR FURTHER WORK:

1. An algorithm be evolved which would be insensitive to the initial estimates of the parameters and converges fast to the correct values of the parameters.
2. The modified Newton Raphson method and the D.F.P. method should be used to extract the stability and control derivatives of high performance aircraft and the wide bodied transport aircraft, to confirm that these methods could be applied to common configurations of aircraft.
3. An effort be made to find out a suitable weighting matrix  $D_1$  to avoid divergence after converging to the minima in case of the modified Newton Raphson method.

The effect of neglecting the second term in Eq. (2.18) be explored to find out its effect on the overall results of the modified Newton Raphson technique.

4. An effort be made to modify the D.F.P. method suitably to reduce the computation time.

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APPENDIX AEQUATIONS OF MOTION:

The lateral-directional equations of motion of an airplane are (21):

$$-L_{\beta} \beta + \dot{p} - L_p p - \frac{I_{xz}}{I_{xx}} \dot{r} - L_r r = L_{\delta_a} \delta_a + L_{\delta_r} \delta_r \quad (A.1)$$

$$-N_{\beta} \beta + \frac{I_{xz}}{I_{zz}} \dot{p} - N_p p + \dot{r} - N_r r = N_{\delta_a} \delta_a + N_{\delta_r} \delta_r \quad (A.2)$$

$$\dot{\beta} - Y_{\beta} \beta - Y_p p - \frac{g}{V} \phi + r - Y_r r = Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r \quad (A.3)$$

Neglecting the product of inertia  $I_{xz}$  and assuming  $y_r = 0$ , these equations reduce to the following:

$$\dot{p} = L_p p + L_r r + L_{\beta} \beta + L_{\delta_a} \delta_a + L_{\delta_r} \delta_r + L_o \quad (A.4)$$

$$\dot{r} = N_p p + N_r r + N_{\beta} \beta + N_{\delta_a} \delta_a + N_{\delta_r} \delta_r + N_o \quad (A.5)$$

$$\dot{\beta} = Y_p p - r + Y_{\beta} \beta + Y_{\phi} \phi + Y_{\delta_a} \delta_a + Y_{\delta_r} \delta_r + Y_o \quad (A.6)$$

where  $Y_{\phi} = \frac{g}{V}$  and  $L_o$ ,  $N_o$ ,  $Y_o$  are the effect of uncertain bias on the measurement of  $\dot{p}$ ,  $\dot{r}$  and  $\dot{\beta}$ .

Realizing that  $\dot{\phi} = p$ . Eq. (A.4 - A.6) when written in the matrix form become

$$\begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix} \quad (4.7)$$

Equation (4.7) can be represented as

$$\dot{x}(t) = A x(t) + B u(t) \quad (4.8)$$

where

$$x = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{p} \\ \dot{r} \\ \dot{\beta} \\ \dot{\phi} \end{bmatrix} \quad u = \begin{bmatrix} \delta_a \\ \delta_r \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} L_p & L_r & L_\beta & 0 \\ N_p & N_r & N_\beta & 0 \\ Y_p & -1 & Y_\beta & Y_\phi \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} L_{\delta_a} & L_{\delta_r} & L_o \\ N_{\delta_a} & N_{\delta_r} & N_o \\ Y_{\delta_a} & Y_{\delta_r} & Y_o \\ 0 & 0 & 0 \end{bmatrix}$$

Note that  $x$ ,  $\dot{x}$  and  $u$  are time varying while the elements of matrices  $A$  and  $B$  are constant.

APPENDIX BDETERMINATION OF ELEMENTS OF  $\nabla_c x$ :

The elements of gradient  $x$  can be determined in the following manner.

Differentiating state Eq. (2.8) with respect to  $a_{ij}$ , we obtain

$$\begin{aligned} \frac{\partial \dot{x}}{\partial a_{ij}} &= \frac{\partial A}{\partial a_{ij}} x + A \frac{\partial x}{\partial a_{ij}} + \cancel{\frac{\partial B}{\partial a_{ij}} u} + B \cancel{\frac{\partial u}{\partial a_{ij}}} \\ &= A \frac{\partial x}{\partial a_{ij}} + \frac{\partial A}{\partial a_{ij}} x \end{aligned} \quad (B.1)$$

By solving the differential equation (B.1), we obtain for  $\frac{\partial x}{\partial a_{ij}}$ .

$$\frac{\partial x}{\partial a_{ij}} = \int_0^t e^{A(t-\tau)} A_{a_{ij}} x(\tau) d\tau$$

Again for coefficients in  $B$ , we obtain

$$\frac{\partial x}{\partial b_{ij}} = \int_0^t e^{B(t-\tau)} B_{b_{ij}} u(\tau) d\tau$$

Similarly if the change with respect to initial conditions  $x_i(0)$  is considered, we obtain

$$\frac{\partial x}{\partial x_i(0)} = e^{At} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix}$$

where 1 appears in  $i^{\text{th}}$  row.

# APPENDIX C

## LISTING OF MODIFIED NEWTON-RAPHSON PROGRAMME

```

IEJOB
IBFTC  MAIN

PROGRAMM  VARIABLES

A  SYSTEM  MATRIX
R  CONTROL MATRIX
X  MEASURED RESPONSE
U  CONTROL DATA
CDX CALCULATED RESPONSE
ITR  NUMBER OF ITERATIONS
F  STATE TRANSFORMATION MATRIX
DEL TIME INCREMENT
TI  ACCORD LENGTH
N  NUMBER OF TIME POINTS
D  WEIGHTING MATRIX FOR MEASURED RESPONSE
H  GRADIENT OF CALCULATED RESPONSE VARIABLE W.R.T.
PARAMETER VECTOR
NOPS  NUMBER OF PROBLEMS TO BE SOLVED
DX  DERIVATIVE OF RESPONSE VARIABLE
Z  DERIVATIVE OF COST FUNCTIONAL W.R.T. PARAMETER VECTOR
AF  SECOND DERIVATIVE OF COST FUNCTIONAL W.R.T.
PARAMETER VECTOR

**THIS PROGRAM EVALUATES AIRCRAFT PARAMETERS FROM FLIGHT DATA
DIMENSION X(110,7),U(110,3),A(4,4),B(4,3),DX(110,4),CDX(110,9),
1AY(110),H(110,7,15),Z(110),AF(20,20),TX(20,1)
DIMENSION O(7),F(30)
COMMON/ANT/A,B,X,U
CALL FLUN(30000)
*****
READ INPUT DATA
READLOC,NOPS
00  FORMAT(12)
CALL FLOW(30000)
DO 1 CANCP=1,NOPS
  READ1,TI,A
  FORMAT(F8.4,13)
  DEL=TI/FLOAT(N-1)
  REAC2=((IX(I,J),J=1,4),(U(1,J),J=1,3),I=1,N)
  FORMAT(17F10.3)
  REAC3=((IAT(I,J),J=1,4),I=1,4),((IB(I,J),J=1,3),I=1,4)
  FORMAT(10F8.4)
  PRINT 705
00  FORMAT(10X,'MATRIX &&& IS *')
  PRINT 710,((IAT(I,J),J=1,4),I=1,4)
10  FORMAT(5X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4)
  PRINT 720
20  FORMAT(10X,'MATRIX &&& IS *')
  PRINT 730,((IB(I,J),J=1,3),I=1,4)
30  FORMAT(5X,F8.4,5X,F8.4,5X,F8.4)
*****
ITERATION LOOP STARTS
ITR=1
*****
CALCULATE DERIVATIVES OF RESPONSE VARIABLES
DO 2 J=1,N
  DO 3 K=1,4
    IF(J.GT.1)GOTO 4
    DX(J,K)=(X(J+1,K)-X(J,K))/DEL
    GOTO 5
    IF(J.EQ.N)GOTO 3
    DX(J,K)=(X(J+1,K)-X(J-1,K))/DEL*0.5
    GOTO 5
    CX(J,K)=(X(J,K)-X(J-1,K))/DEL
  CONTINUE
  DO 3 J=1,N

```

```

DC 500 I=1,N
X(I,J,K)=0.0 (J,K=1)
PRINT 100
) FORMAT(1/2X, (O1 H*)/12X, *TIME*, 2H*, *CONTROL VARIABLE*, 1H*, *
IF S P N S E V A P I B L S*/
5X, O1 H*/ 2X, *S*, 2X, *AILERON*, 1X, *RUDDER*, 12X, *ROLL*, 6X, *Y
5X, *X*, 10X, *CLIP*, 2X, *ROLL*, 7X, *ROLL*, 5X, *YAW*, 2X, *SIDE SLIP*, 7X, *
4X, *ANGLE*, 6X, *ANGLE*, 12X, *VEL*, 6X, *VEL*, 4X, *ANGLE*, 6X, *ANGLE*,
5X, *AC*, 6X, *AC*, 6X, *VEL*/20X, *DEG*, 4X, *DEG*, 10X, *DEG/S*, 6X, *
6X, *DEG/S*, 6X, *DEG/S*, 6X, *DEG/S*, 6X, *DEG/S*, 6X, *DEG/S*, 6X, *DEG/S*,
7*/9, *C*(1 H*)
TT=DEL
DC 500 I=1,N
TT=TT+OCL
PRINT 100, TT, (U(I,J), J=1,3), (X(I,J), J=1,7)
FORMAT(10X, F6.2, 2F8.4, F6.3, 7F10.6/)
) *****
CALCULATE COST FUNCTIONAL AND ITS DERIVATIVE DEL C J
CALL CALOIF(CDX,N,DEL,H)
DO 6 J=1,15
FILTR)=0.0
Z(J)=0.
DC 6 K=1,N
DO 6 JK=1,7
ZX=(Z-Y) DIFFERENCE OF RESPONCE
IF(JK.GT.4) GO TO 7
ZX=X(K,JK)-CDX(K,JK)
IF(K.EQ.1.OR.K.EQ.N) ZX=ZX/2.
GO TO 6
ZX=DX(K,JK-4)-CDX(K,JK)
IF(K.EQ.1.OR.K.EQ.N) ZX=ZX/2.0
GO TO 6
F(ITR)=F(ITR)+ZX*ZX
Z(J)=Z(J)+H(K,JK,J)*ZX
PRINT 500, F(ITR)
FORMAT(5X, *VALUE OF FIT ERROR IS *, E15.7)
IF(ITR.EQ.1) GOTO 600
IF(F(ITR).GT.F(ITR-1)) GO TO 90
CONTINUE
) *****
CALCULATE DEL SQ. C J
DC 10 J=1,20
DC 10 K=1,20
AF(J,K)=C.
DO 10 JK=1,N
XNO=1.0
IF(JK.EQ.1.OR.JK.EQ.N) XNO=0.5
DO 10 J=1,15
DO 10 K=1,15
DC 10 KJ=1,7
AF(J,K)=AF(J,K)+H(JK,KJ,J)*H(JK,KJ,K)*XNO
DC 100 J=1,15
Z(J)=Z(J)+DEL
DC 100 K=1,15
AF(J,K)=AF(J,K)+DEL
) *****
CALCULATE DELTA C
DO 15 J=1,20
15 TX(J,1)=0.0
CALL MATINV(AF,15, TX,1)
DO 15 J=1,15
TX(J,1)=0.0
DO 15 K=1,15
TX(J,1)=AF(J,K)*Z(K)+TX(J,1)
) *****

```

```

DE7K=1,7
CDX(J,K)=CZ(J-1,K)+CDX(J-1,K+4)*DEL
DC1K=1,4
CDX(J,K+4)=C
DC9JK=1,7
CDX(J,K+4)=CDX(J,K+4)+C(K,JK)*CDX(J,JK)
DC10JK=1,7
CDX(J,K+4)=CDX(J,K+4)+B(K,JK)*U(J,JK)
CONTINUE
*****
FORMULATE MATRICES F AND F
DC1J=1,7
DC15K=1,4
IF(J.GT.4)GOTO16
F(J,K)=0.
IF(J.EQ.K)F(J,K)=1.
GOTO15
F(J,K)=A(J-4,K)
CONTINUE
DC17J=1,7
DC17K=1,3
IF(J.GT.4)GOTO18
F(J,K)=0.
GOTO17
F(J,K)=B(J-4,K)
CONTINUE
CALCULATE DEL C (Z-Y)
DO19 J=1,15
DO19K=1,N
DO19JK=1,7
H(K,JK,J)=0.
IDENTIFY ELEMENTS OF C IN MATRICES A AND B
DO20 J=1,15
J1=J
IF(J.GT.8)J1=J-8
IF(J.GT.8)GOTO21
J1=0
IF(J.GT.5)J1=1
IF(J.GT.6)J1=2
J1=J-B+J1
IF((J.EQ.2).AND.(J1.EQ.2))J1=3
J2=J1+1
GOTO22
IF(J1.GT.8)J1=2
IF(J1.GT.6)J1=3
J1=J1+(J1-1)
IF(J1.EQ.3)J1=8
DO20K=1,N
DO20CCK=1,7
IF(J.GT.8)GOTO23
IF(JK.EQ.(4+J2))H(K,JK,J)=H(K,JK,J)-CDX(K,J1)
GOTO200
IF(JK.EQ.(4+J1))H(K,JK,J)=H(K,JK,J)-U(K,J1)
CONTINUE
IF(K.EQ.1)GO TO 20
DO 20 KJ1=1,4
Z(KJ1)=0.C
DC26 KJ=1,4
Z(KJ1)=Z(KJ1)+A(KJ1,KJ)+H(K-1,KJ,J)
DC 27 KJ=1,4
Z(KJ1)=Z(KJ1)+DEL+H(K-1,KJ,J)
IF(J.GT.8)GOTO25
Z(J1)=Z(J1)+DEL+CDX(K-1,J1)
GO TO 24

```

```

34 Z(I,J)=Z(I,J)+EL*U(K-1,I)
DO 1 JK=1,N
DO 1 KJ=1,N
31 H(K,JK,KJ)=H(K,JK,J)+(JK,KJ)*Z(KJ)
20 CONTINUE
RETURN
END

```

```

FTC SUB 2
SUBROUTINE MATINV(A,N,P,LP)
THIS SUBROUTINE FINDS INVERSE OF A MATRIX
A IS INPUT MATRIX(N*N).ON RETURN IT GIVES INVERSE OF A
P IS MATRIX OF UNKNOWN VECTOR OF ORDER N*LP
DIMENSION A(20,20),P(20,1),J1(20),J2(20),NX(20)
DO 44 J1=1,N
GRET=C.
DO 44 M=1,N
IF(J1.EQ.1) GO TO 15
J2=J1-1
DO 34 M1=1,J1
JJ=J1(M1)
IF(N.EQ.JJ) GO TO 44
CONTINUE
DO 45 K=1,N
IF(J1.EQ.1) GO TO 39
DO 35 K1=1,J1
JK=J2(K1)
IF(K.EQ.JK) GO TO 45
CONTINUE
IF(ABS(A(M,K)).LT.ABS(GRET)) GO TO 45
GRET=A(M,K)
UN(J1)=K
UN(J2)=K
CONTINUE
CONTINUE
JA=J2(J1)
JB=J1(J1)
NLP=N+LP
DO 22 M=1,N
Y=A(M,JA)
IF(M.EQ.JB) GO TO 222
DO 220 K=1,NLP
IF(K.GT.N) GO TO 230
A(M,K)=A(M,K)-A(JB,K)*Y/GRET
GO TO 220
KL=K-N
P(M,KL)=P(M,KL)-P(JB,KL)*Y/GRET
CONTINUE
A(M,JA)=Y/GRET
CONTINUE
DO 221 M=1,NLP
IF(M.GT.N) GO TO 231
A(JB,M)=A(JB,M)/GRET
GO TO 221
ML=M-N
P(JB,ML)=P(JB,ML)/GRET
CONTINUE
A(JB,JA)=1./GRET
CONTINUE
DO 800 KK=1,N
DO 802 J=1,N

```

```

JK=J1(J)
JC=J2(J)
IF(KK.EC.JC) GO TO 804
CONTINUE
NX(KK)=JK
CONTINUE
IF(JK.LT.KK) JK=NX(JK)
IF(JK.LT.KK) GO TO 806
DO FOR JS=1,NLP
IF(JS.GT.N) GO TO 810
Z=A(KK,JS)
A(KK,JS)=A(JK,JS)
A(JK,JS)=Z
GO TO 806
KKL=JS+N
Z=P(KK,KKL)
P(KK,KKL)=P(JK,KKL)
P(JK,KKL)=Z
CONTINUE
CONTINUE
DO 43 MN=1,N
DO 40 NN=1,N
JK=J1(NN)
JC=J2(NN)
IF(MA.EC.JK) GO TO 25
CONTINUE
NX(MN)=JC
CONTINUE
IF(JC.LT.MN) JC=NX(JC)
IF(JC.LT.MN) GO TO 1
DO 53 HM=1,N
Z=A(MN,JK)
A(MN,JK)=A(MN,JC)
A(MN,JC)=Z
RETURN
END

```

Y

# APPENDIX D

## 16 OF DAVIDSON-FLETCHER-POWELL PROGRAMME

```

BJOB
BFTC  MAIN
      DESCRIPTION OF PARAMETERS.
A     SYSTEM MATRIX
B     CONTROL MATRIX
X     MEASURED RESPONSE
U     CONTROL DATA
CDX   CALCULATED RESPONSE
DEL   TIME INCREMENT
TI     HECCPD LENGTH
N      NUMBER OF TIME POINTS
H      GRADIENT OF CALCULATED RESPONSE VARIABLE W.R.T.
      PARAMETER VECTOR
NOPS   NUMBER OF PROBLEMS TO BE SOLVED
DX     DERIVATIVE OF RESPONSE VARIABLE
**THIS PROGRAM EVALUATES AIRCRAFT PARAMETERS FROM FLIGHT DATA
DIMENSION X(110,7),U(110,3),A(4,4),B(4,3),DX(110,4),
1CDX(110,4),C(115),G(15)
COMMON A,B,X,U,N,DEL
DATA EPS,LIMIT,NA/0.5,40,15/
DATA EST/70./
M=(NN*(NN+7))/2
READ100,ACPS
FORMAT(112)
DC(CINOP=1,NOPS
  READ1,TI,A
  FORMAT(F8.4,13)
  DEL=TI/FLCAT(N-1)
  READ2,((X(I,J),J=1,4),(U(I,J),J=1,3),I=1,N)
  FORMAT(17F10.6)
  READ3,((A(I,J),J=1,4),I=1,4),((B(I,J),J=1,3),I=1,4)
  FORMAT(8F10.6)
  PRINT 700
  FORMAT(10X,'MATRIX A IS *')
  PRINT 710,((A(I,J),J=1,4),I=1,4)
  FORMAT(5X,F8.4,5X,F8.4,5X,F8.4,5X,F8.4)
  PRINT 720
  FORMAT(10X,'MATRIX B IS *')
  PRINT 730,((B(I,J),J=1,3),I=1,4)
  FORMAT(5X,F8.4,5X,F8.4,5X,F8.4)
  ITR=1
  DO3 J=1,N
    DO4 K=1,4
      IF(IJ.GT.1)GOTO4
      OX(J,K)=(X(J+1,K)-X(J,K))/DEL
      GOTO3
      IF(IJ.EC.N)GOTO5
      OX(J,K)=(X(J+1,K)-X(J-1,K))/DEL*0.5
      GOTO3
      CX(J,K)=(X(J,K)-X(J-1,K))/DEL
      CONTINUE
      CALL FLUN(BCC00)
      PRINT750,((X(I,J),J=1,4),(DX(I,J),J=1,4),I=1,N)
      FORMAT(5X,8F10.6)
      CALL PLOV(BCC00)
      DO7C I=1,N
        DO7C J=5,7
          X(I,J)=OX(I,J-4)
          IDENTIFY UNKNOWN PARAMETERS IN MATRIX A AND B
          DO7C I=1,N
            IF(I.GT.1)GOTO10
            C(I)=A(1,I)
            GO7C 80
            IF(I.GT.5) GOTO 20
            C(I)=A(2,I-5)

```

```

GO TO 30
IF(I.GT.8) GOTO 20
C(I)=A(I,1)
C(I)=A(I,2)
GO TO 40
IF(I.GT.12) GO TO 40
C(I)=B(I,1)
GO TO 50
IF(I.GT.14) GOTO 50
C(I)=B(I,1)
GO TO 60
C(I)=B(I,2)
CONTINUE
PRINT 300, (C(I), I=1,15)
CALL FUNCT(N,C,F,G)
PRINT 920, F
FORMAT(5X, E14.6)
CALL DEFP(N,N,C,F,EST,EPS,LIMIT,IER,KOUNT)
PRINT 300, (C(I), I=1,15)
FORMAT(5X, 8(2X, F12.6))
PRINT 310, IER, KOUNT, F
FORMAT(5X, 13, 7X, 12, 7X, E14.6)
CALL FUNCT(N,C,F,G)
PRINT 700
PRINT 710, ((A(I,J), J=1,4), I=1,4)
PRINT 720
PRINT 730, ((B(I,J), J=1,3), I=1,4)
CALL CALDIFF(CDX,H)
PRINT 995
PRINT 860
FORMAT(/9X, 100(1H=)/12X, *TIME*, 1H=, *CONTROL VARIABLES*, 1H=, *R
E S P O N S E V A R I A B L E S*/
9X, 100(1H=)/12X, *SEC*, 2X, *AILERON*, 1X, *RUDDER*, 12X, *ROLL*, 6X, *YA
W*, 4X, *SIDE SLIP*, 2X, *ROLL*, 7X, *ROLL*, 5X, *YAW*, 2X, *SIDE SLIP*, 7, 19
X, *ANGLE*, 3X, *ANGLE*, 12X, *VEL*, 6X, *VEL*, 4X, *ANGLE*, 6X, *ANGLE*, 6X,
*ACC*, 5X, *ACC*, 6X, *VEL*/20X, *DEG*, 4X, *DEG*, 10X, *DEG/SEC*, 2X,
*DEG/SEC*, 4X, *DEG*, 3X, *DEG*, 3X, *DEG/SEC*, 3X, *DEG/SEC*
/9X, 100(1H=))
TT=DEL
DO 570 I=1,N
TT=TT+DEL
PRINT 770, TT, ((U(I,J), J=1,3), (CDX(I,J), J=1,7))
FORMAT(9X, 1H=, F6.2, 2F8.4, F6.2, 4F10.6, F11.6, 2F10.6, 1H=/)
FORMAT(13H=)
PRINT 999
CONTINUE
STOP
END

```

# SUB 1

```

SUBROUTINE DEFP(N,M,X,F,EST,EPS,LIMIT,IER,KOUNT)
N =NUMBER OF INDEPENDENT VARIABLES
X =INDEPENDENT VARIABLE VECTOR (INITIAL VALUES ON INPUT,
OPTIMUM VALUES ON OUTPUT)
F =FINAL MINIMUM VALUE OF THE OBJECTIVE FUNCTION
G =FINAL GRADIENT VECTOR AT THE MINIMUM
H =STORAGE VECTOR
M =STORAGE VECTOR DIMENSION (M=(N+7))/2
EST =ESTIMATE OF THE MINIMUM VALUE OF THE OBJECTIVE FUNCTION
EPS =TEST VALUE REPRESENTING THE EXPECTED ABSOLUTE ERROR IN

```

```

MOVEMENT
LIMIT = MAXIMUM NUMBER OF ITERATIONS
IER = STOPPING PARAMETER
IER=0 MEANS CONVERGENCE WAS OBTAINED
IER=1 MEANS NO CONVERGENCE IN LIMIT ITERATIONS
IER=2 MEANS ERRORS IN GRADIENT CALCULATION
IER=3 MEANS IT IS LIKELY THAT A MINIMUM DOES NOT EXIST
KOUNT = ITERATION COUNTER

```

```

DIMENSION H(16), X(5), G(15), A(4,4), B(4,3), D(110,7), U(110,3)
COMMON A, H, D, U, I, DEL

```

```

COMPUTE FUNCTION VALUE AND GRADIENT VECTOR FOR INITIAL ARGUMENT
CALL FUNCT(N,X,F,G)

```

```

RESET ITERATION COUNTER AND GENERATE IDENTITY MATRIX

```

```

IER=0
KOUNT=0
N2=N+N
N3=N2+N
N31=N3+1
K=N31
DO 4 J=1,N
  H(K)=1.
  NJ=N-J
  IF(NJ) 5,5,2
  DO 3 L=1,NJ
    KL=K+L
    H(KL)=0.
    K=KL+1
  
```

```

START ITERATION LOOP

```

```

KOUNT=KOUNT+1
KONT=KOUNT-1
PRINT 900,KONT,(X(I),I=1,15)

```

```

SAVE FUNCTION VALUE, ARGUMENT VECTOR AND GRADIENT VECTOR

```

```

CALL FLUN(30000)
CALL FLOV(30000)
OLDP=F
DO 9 J=1,N
  K=N+J
  H(K)=G(J)
  K=K+N
  H(K)=X(J)

```

```

DETERMINE DIRECTION VECTOR H.

```

```

K=J+N3
T=0.
DO 8 L=1,N
  T=T-G(L)*H(K)
  IF(L-J) 6,7,7
  K=K+N-L
  GO TO 8
  K=K+1
CONTINUE
H(J)=T

```

```

CHECK WHETHER FUNCTION WILL DECREASE STEPPING ALONG H.

```

```

DY=0.
HNRN=0.
GNRN=C.

```

```

CALCULATE DIRECTIONAL DERIVATIVE AND TEST VALUES FOR DIRECTION
VECTOR H AND GRADIENT VECTOR G.

```

```

DO 10 J=1,N
HNRN=HNRN+H*PSIG(J)
GNRN=GNRN+G*PSIG(J)
DY=LY+H(J)*G(J)

REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTIONAL
DERIVATIVE APPEARS TO BE POSITIVE OR ZERO
IF(DY) 11,*,11

REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF DIRECTION
VECTOR H IS SMALL COMPARED TO GRADIENT VECTOR G.
IF(HNRN/GNRN*EPS) 5,51,12

SEARCH MINIMUM ALONG DIRECTION H

SEARCH ALONG H FOR POSITIVE DIRECTIONAL DERIVATIVE
FY=F
ALFA=2.*(F1-F)/DY
AMBOA=L.

USE ESTIMATE FOR STEPSIZE ONLY IF IT IS POSITIVE AND LESS THAN
1. OTHERWISE TAKE 1. AS STEP SIZE
IF(ALFA) 15,15,12
IF(ALFA-AMBOA) 14,15,15
AMBOA=ALFA
ALFA=C.

SAVE FUNCTION AND DERIVATIVE VALUES FOR OLD ARGUMENT
FX=FY
DX=DY

STEP ARGUMENT ALONG H
DO 17 I=1,N
X(I)=X(I)+AMBOA*H(I)

COMPUTE FUNCTION VALUE AND GRADIENT FOR NEW ARGUMENT
CALL FUNCTIN,X,F,G)
FY=F

COMPUTE DIRECTIONAL DERIVATIVE DY FOR NEW ARGUMENT, TERMINATE
SEARCH, IF DY IS POSITIVE. IF DY IS ZERO THE MINIMUM IS FOUND
DY=C.
DO 18 I=1,N
DY=DY+G(I)*H(I)
IF(DY) 19,20,22

TERMINATE SEARCH ALSO IF THE FUNCTION VALUE INDICATES THAT
A MINIMUM HAS BEEN PASSED
IF(FY-FX) 20,22,22

REPEAT SEARCH AND DOUBLE STEPSIZE FOR FURTHER SEARCHES
AMBOA=AMBOA+ALFA
ALFA=AMBOA
END OF SEARCH LOOP

TERMINATE IF THE CHANGE IN ARGUMENT GETS VERY LARGE
IF(HNRN*AMBOA-1.) 10, 16,16,23

LINEAR SEARCH TECHNIQUE INDICATES THAT NO MINIMUM EXISTS
IER=2
RETURN

INTERPOLATE CLERICALLY IN THE INTERVAL DEFINED BY THE SEARCH
ABOVE AND COMPUTE THE ARGUMENT X FOR WHICH THE INTERPOLATION
POLYNOMIAL IS MINIMIZED

```

```

T=0.
IF(AMBOA) 27,28,24
Z=3.*(PX-FY)/AMBOA+EX+DY
ALFA=AMAX(1,PS(Z),ABS(DX),ABS(DY))
DALFA=Z/ALFA
DALFA=DALFA+DALFA-DY/ALFA+DY/ALFA
IF(CALFA) 31,32,35
W=ALFA*SQRT(CALFA)
ALFA=(DY+W-Z)*AMBOA/(DY+2.*W-DX)
DO 26 I=1,N
X(I)=X(I)+(T*ALFA)*H(I)

```

TERMINATE, IF THE VALUE OF THE ACTUAL FUNCTION AT X IS LESS THAN THE FUNCTION VALUES AT THE INTERVAL ENDS. OTHERWISE REDUCE THE INTERVAL BY CHOOSING ONE-END POINT EQUAL TO X AND REPEAT THE INTERPOLATION. WHICH END POINT IS CHOSEN DEPENDS ON THE VALUE OF THE FUNCTION AND ITS GRADIENT AT X

```

CALL FUNCTIN,X,F,G)
IF(F-FX) 27,27,28
IF(F-FY) 26,36,28
DALFA=0.
DO 29 I=1,N
DALFA=DALFA+G(I)*H(I)
IF(CALFA) 30,33,33
IF(F-FX) 32,31,33
IF(DX-DALFA) 32,36,32
FX=F
DX=DALFA
T=ALFA
AMBOA=ALFA
GO TO 23
IF(FY-F) 35,34,35
IF(DY-DALFA) 35,36,35
FY=F
DY=DALFA
AMBOA=AMBOA-ALFA
GO TO 22

```

COMPUTE DIFFERENCE VECTORS OF ARGUMENT AND GRADIENT FROM TWO CONSECUTIVE ITERATIONS

```

DO 37 J=1,N
K=N+J
H(K)=G(J)-H(K)
K=N+K
H(K)=X(J)-H(K)
IF(KOUNT.EQ.2) GOTO 60
IF(ABS(OLDF-F).LE.EP) GOTO 55
CONTINUE

```

TERMINATE, IF FUNCTION HAS NOT DECREASED DURING LAST ITERATION

```

IF(OLDF-F+EPS) 51,58,38

```

TEST LENGTH OF ARGUMENT DIFFERENCE VECTOR AND DIRECTION VECTOR. IF AT LEAST N ITERATIONS HAVE BEEN EXECUTED. TERMINATE, IF BOTH ARE LESS THAN EPS

```

IER=0
IF(KOUNT-N) 42,39,39
T=0.
Z=0.
DO 40 J=1,N
K=N+J
H(K)=H(K)
K=N+K
T=T+ABS(H(K))

```

```

Z=Z+W*H(K)
IF (GNRM-EPS) 47,47,42
IF (1-EPS) 47,50,42

```

```

TERMINAT, IF NUMBER OF ITERATIONS WOULD EXCEED LIMIT
IF (KOUNT-LIMIT) 47,50,50

```

```

PREPARE UPDATING OF MATRIX H
ALFA=C.
DO 47 J=1,N
K=J+N
W=0.
DO 46 L=1,N
KL=N+L
W=W+H(KL)*H(K)
IF (L-J) 46,46,45
K=K+N-L
GO TO 46
K=K+1
CONTINUE
K=N+J
ALFA=ALFA+W*H(K)
H(J)=W

```

```

REPEAT SEARCH IN DIRECTION OF STEEPEST DESCENT IF RESULTS
ARE NOT SATISFACTORY
IF (Z=ALFA) 48,1,48

```

```

UPDATE MATRIX H
K=N+1
DO 49 L=1,N
KL=N+L
DO 49 J=L,N
NJ=N+J
H(K)=H(K)+H(KL)*H(NJ)/Z-H(L)*H(J)/ALFA
K=K+1
PRINT900, KOUNT, IX(1), I=1,15)
FORMAT(1X,13,7(2X,F12.6))
PRINT910, F
FORMAT(2X,=VALUE OF F IS*,E14.6)
GO TO 5
END OF ITERATION LOOP

```

```

NO CONVERGENCE AFTER LIMIT ITERATIONS
IER=1
RETURN

```

```

RESTORE OLD VALUES OF FUNCTION AND ARGUMENTS
DO 52 J=1,N
K=N+J
X(J)=H(K)
CALL FUNCTION(X,F,G)

```

```

REPEAT IN DIRECTION OF STEEPEST DESCENT IF DERIVATIVE
FAILS TO BE SUFFICIENTLY SMALL
IF (GNRM-EPS) 55,55,52

```

```

TEST FOR REPEATED FAILURE OF ITERATION
IF (IER) 56,54,54
IER=-1
GO TO 1
IER=0
RETURN
END

```

TC SLB 2

SUBROUTINE FUNCT(I,C,F,G)

ARGUMENT LIST

I = NO. OF INDEPENDENT VARIABLES

C = VECTOR OF X VALUES

F = OBJECTIVE FUNCTION EQUATION.

G = VECTOR OF OBJECTIVE FUNCTION DERIVATIVES, (N LONG).

COMMONA,B,X,U,N,DEL

DIMENSION C(15),G(15),A(4,4),B(4,3),X(110,7),U(110,3),CDX(10,8),

1H(1,0,7,15)

DO 30 I=1,3

DO 30 J=1,3

IF(I.GT.1) GOTO 10

A(I,J)=C(J)

B(I,J)=C(J+8)

GO TO 30

IF(I.GT.2) GOTO 20

A(I,J)=C(J+3)

B(I,J)=C(J+11)

GO TO 30

A(3,1)=C(7)

A(3,3)=C(8)

B(3,3)=C(15)

CONTINUE

CALL CALDIF(CDX,H)

DO 6 J=1,15

G(J)=0.0

F=0.0

DO 6 K=1,N

DO 6 JK=1,7

ZX=X(K,JK)-CDX(K,JK)

IF(K.EQ.1.OR.K.EQ.N) ZX=ZX/2.

ZX=ZX\*10.E-02

F=F+ZX\*ZX

G(J)=G(J)+H(K,JK,J)\*ZX

RETURN

END

TC SLB 3

SUBROUTINE CALDIF(CDX,H)

SUBROUTINE CALDIF CALCULATES RESPONSE AND GRADIENT OF RESPONSE  
W.R.T. UNKNOWN PARAMETERS

COMMONA,B,X,U,N,DEL

DIMENSION C(4,4),E(7,4),F(7,4),H(110,7,15),X(110,7),U(110,3),

1A(4,4),B(4,3),CDX(110,8),Z(110)

DO 300 I=1,4

DO 300 J=1,4

C(I,J)=A(I,J)

DO 3 J=1,4

CDX(I,J)=X(I,J)

DO 2 J=1,4

CDX(I,J+4)=0.

DO 4 K=1,6

CDX(I,J+4)=CDX(I,J+4)+C(J,K)\*CDX(I,K)

DO 4 K=1,7

CDX(I,J+4)=CDX(I,J+4)+B(J,K)\*U(I,K)

```

CONTINUE
DC1J=1,4
DC1K=1,4
CDX(J,K)=C(J-1,K)+CDX(J-1,K+4)*DEL
DC1K=1,4
CC1(J,K+4)=C.
DC1JK=1,4
CDX(J,K+4)=CDX(J,K+4)+C(K,JK)*CDX(J,JK)
DC1JK=1,4
CDX(J,K+4)=CDX(J,K+4)+B(K,JK)*U(J,JK)
CONTINUE
DC15J=1,7
DC15K=1,4
IF(J.GT.4)GOTO16
E(J,K)=C.
IF(J-Q.K)E(J,K)=C.
GOTO15
E(J,K)=A(J-4,K)
CONTINUE
DC17J=1,7
DC17K=1,3
IF(J.GT.4)GOTO18
F(J,K)=0.
GOTO17
F(J,K)=B(J-4,K)
CONTINUE
DO19 J=1,15
DO19K=1,N
DO19JK=1,7
H(K,JK,J)=0.
DO20 J=1,15
J=J
IF(J.GT.8)J=J-8
IF(J.GT.8)GOTO21
IJ=0
IF(J.GT.3)IJ=1
IF(J.GT.6)IJ=2
IJ=J-3-IJ
IF(IJ.EQ.2).AND.(I2.EQ.2))I1=3
I2=IJ+1
GOTO22
IJ=1
IF(IJ.GT.3)IJ=2
IF(IJ.GT.4)IJ=3
I1=IJ-3-(IJ-1)
IF(I1.EQ.3)I1=3
DO23K=1,N
DO23CCK=1,7
IF(IJ.GT.8)GOTO24
IF(IJK.EQ.(4+I2))H(K,JK,J)=H(K,JK,J)-CDX(K,I1)
GOTO23C
IF(IJK.EQ.(4+I1))H(K,JK,J)=H(K,JK,J)-U(K,I1)
CONTINUE
IF(K.EQ.1)GO TO 20
DO 26 KJ3=1,4
Z(KJ1)=0.C
DO24 KJ=1,4
Z(KJ2)=Z(KJ3)+A(KJ,KJ)*H(K-1,KJ,J)
DO 27 KJ=1,K
Z(KJ)=Z(KJ)+DEL*H(K-1,KJ,J)
IF(IJ.GT.8)GOTO25
Z(I2)=Z(I2)+DEL*CDX(K-1,I1)
GO TO 24
Z(I1)=Z(I1)+DEL*U(K-1,I1)
24 DO 31 JK=1,7

```

```
DO 31 KJ=1,2  
31 HIK,JK,J)=HIK,JK,J)+Z(JK,KJ)*Z(KJ)  
20 CONTINUE  
RETURN  
END  
ENTRY
```

[illegible]

[illegible]



7.00	0.7000	1.00	1.000000	0.109700	6.065600	73.000000	74.217555	-1.454000	2.591500*
7.00	0.7200	1.00	1.000000	0.004800	6.271000	71.023000	74.594555	-0.921000	1.470500*
7.00	0.7400	1.00	1.000000	-0.074500	6.399700	68.301000	74.095555	0.561000	0.171000*
7.00	0.7600	1.00	1.000000	-0.117500	6.445200	64.817000	71.260000	5.151000	0.010000*
7.00	0.7800	1.00	1.000000	-0.132800	6.403700	60.645000	71.100000	0.725700	0.187100*
7.00	0.8000	1.00	1.000000	-0.059750	6.271700	55.746000	73.794555	1.191000	2.061000*
7.00	0.8200	1.00	1.000000	0.906600	5.991400	50.113000	78.184555	16.001000	6.621000*
7.00	0.8400	1.00	1.000000	2.067600	5.459000	42.681000	74.259955	24.110000	6.731000*
7.00	0.8600	1.00	1.000000	4.585600	5.645200	36.497000	59.270000	15.701000	9.221000*
7.00	0.8800	1.00	1.000000	5.981900	3.594700	28.714000	41.460000	12.075500	11.418000*
7.00	0.9000	1.00	1.000000	7.606700	2.361600	20.516000	21.820000	8.136500	12.553000*
7.00	0.9200	1.00	1.000000	7.815200	1.004100	12.099000	-1.334956	4.015000	13.194600*
7.00	0.9400	1.00	1.000000	7.887700	-0.417320	3.668700	19.280000	-0.156500	14.230500*
7.00	0.9600	1.00	1.000000	7.584900	1.842000	-4.568600	39.064555	-4.181000	13.568400*
7.00	0.9800	1.00	1.000000	6.470700	3.211000	12.412000	55.695000	-7.582000	13.166500*
7.00	1.0000	1.00	1.000000	6.067500	4.675300	19.656000	68.710000	-10.258000	11.928500*
7.00	1.0200	1.00	1.000000	4.915100	5.594700	26.291000	79.474955	-12.511000	10.138000*
7.00	1.0400	1.00	1.000000	3.965300	6.542900	32.089000	87.974955	-14.342000	-8.454500*
7.00	1.0600	1.00	1.000000	2.050700	7.287600	37.606000	93.590000	-15.701000	-6.344500*
7.00	1.0800	1.00	1.000000	0.424300	7.911800	40.980000	97.365000	-16.567500	-4.079500*
7.00	1.1000	1.00	1.000000	-1.262800	8.103500	43.978000	98.044998	-16.915500	-1.733000*
7.00	1.1200	1.00	1.000000	-2.958800	8.158400	45.994000	96.037500	-16.750500	0.620000*
7.00	1.1400	1.00	1.000000	-4.612900	7.979500	47.047000	91.509500	-16.050000	2.507000*
7.00	1.1600	1.00	1.000000	-6.174800	7.577000	47.183000	84.587500	-14.966500	5.058000*
7.00	1.1800	1.00	1.000000	-7.606200	6.987900	46.472000	58.600500	-10.405500	4.921500*
7.00	1.2000	1.00	1.000000	-8.257900	6.592700	45.825000	36.640000	-6.517000	3.752000*

[illegible]

[illegible]

4.20	1.000	1.00	23.706234	0.184295	0.315187	15.005163	0.13.737107	2.0.6690	2.569289*
4.30	1.000	1.00	32.182524	0.684964	0.672116	16.45786	19.720350	1.0.7757	1.9.2.48*
4.40	1.000	1.00	30.210484	1.259740	0.969365	21.614038	23.297810	1.0.7757	1.224.49*
4.50	1.000	1.00	27.580599	1.916233	1.198308	27.675080	45.642359	1.0.7757	1.542.37*
4.60	1.000	1.00	25.311462	2.693263	1.352686	27.463156	86.370034	1.0.7757	1.0.3.3.3*
4.70	1.000	1.00	16.472179	4.421727	1.386940	29.794302	101.64965	1.0.7757	1.704.5*
4.80	1.000	1.00	4.308810	6.051027	1.216467	31.201080	82.050145	1.0.7757	1.0.1.3.3*
4.90	1.000	1.00	1.00	7.256204	0.827638	31.712561	52.831007	1.0.7757	1.5.42.309*
5.00	1.000	1.00	2.039305	8.086249	0.281281	31.336940	22.597024	1.0.7757	1.0.3.0.04*
5.10	1.000	1.00	1.99008	8.820729	0.351629	30.453010	10.069710	1.0.7757	1.6.473.02*
5.20	1.000	1.00	1.92037	8.276286	0.998945	29.313109	41.185852	1.0.7757	1.5.859522*
5.30	1.000	1.00	1.87352	7.652489	1.588937	28.293905	70.801567	1.0.7757	1.4.643.007*
5.40	1.000	1.00	1.006705	6.590220	2.053298	27.686559	98.033466	1.0.7757	1.2.760724*
5.50	1.000	1.00	1.00051	5.111323	2.329374	27.787230	111.576719	1.0.7757	1.0.328.14*
5.60	1.000	1.00	21.947923	3.422479	2.362245	28.808235	111.020909	1.0.7757	1.2.330236*
5.70	1.000	1.00	33.070014	1.134357	2.129221	31.065027	96.386186	1.0.7757	1.4.851.701*
5.80	1.000	1.00	12.704037	0.259375	1.640051	34.372028	68.135985	1.0.7757	1.7.040222*
5.90	1.000	1.00	49.522231	0.796433	0.935989	38.642891	37.660174	1.0.7757	1.6.482.74*
6.00	1.000	1.00	35.288148	1.387173	0.087771	43.595114	5.960310	1.0.7757	1.9.178590*
6.10	1.000	1.00	59.884278	1.465476	0.890128	48.923939	25.934790	1.0.7757	1.9.120.39*
6.20	1.000	1.00	51.290799	1.083067	1.742201	54.312366	57.006020	1.0.7757	1.8.325.85*
6.30	1.000	1.00	45.290197	0.190695	2.574590	59.441446	71.694534	1.0.7757	1.6.850516*
6.40	1.000	1.00	38.420183	0.531216	3.257642	64.000465	69.385673	1.0.7757	1.5.450453*
6.50	1.000	1.00	31.482176	0.416718	3.802687	67.842539	70.067245	1.0.7757	1.4.961766*
6.60	1.000	1.00	24.475451	0.053725	4.298864	70.990756	74.935364	1.0.7757	1.4.802.79*
6.70	1.000	1.00	16.982093	0.385869	4.779061	73.438301	79.460906	1.0.7757	1.4.422.27*
6.80	1.000	1.00	9.036566	0.586210	5.221304	75.136510	83.420177	1.0.7757	1.3.841.98*
6.90	1.000	1.00	0.091997	0.666626	5.605503	76.039910	86.495913	1.0.7757	1.3.088032*



[illegible]

1.90	0.2600	-0.1000	1.00	-7.149369	-0.833350	1.764444	4.173424	1.000000	1.000000
2.00	0.1855	-0.1000	1.00	-4.125165	0.610140	1.732471	3.471920	1.000000	1.000000
2.10	0.1110	-0.1000	1.00	-10.521111	0.444082	1.650646	2.953180	1.000000	1.000000
2.20	0.1100	-0.1000	1.00	-12.160657	1.366732	1.448921	2.408490	1.000000	1.000000
2.30	0.1000	-0.1000	1.00	-14.360081	2.466294	1.180225	0.112421	1.000000	1.000000
2.40	0.1000	-0.1000	1.00	-15.143162	2.721125	0.626024	-1.243583	0.907219	1.000000
2.50	0.1000	-0.1000	1.00	-16.133413	3.220113	0.403805	-2.757645	-1.415545	1.000000
2.60	0.1000	-0.1000	1.00	-16.275221	3.446334	-0.070809	-4.421247	4.414527	2.649111
2.70	0.1000	-0.1000	1.00	-15.951342	3.444466	-0.564098	-6.058751	10.365221	1.000000
2.80	0.1000	-0.1000	1.00	-14.997620	4.097796	-1.136848	-7.652105	16.213789	0.212361
2.90	0.1000	-0.1000	1.00	-13.275141	4.121033	-1.650462	-9.141807	21.748057	-1.019244
3.00	0.1000	-0.1000	1.00	-11.100805	4.015167	-2.168574	-10.469371	26.771093	-2.129843
3.10	0.1000	-0.1000	1.00	-8.402736	3.397122	-2.449335	-11.579454	31.107750	-3.211357
3.20	0.1000	-0.1000	1.00	-5.717347	3.475962	-3.081565	-12.421827	34.610178	-4.051607
3.30	0.1000	-0.1000	1.00	-3.341929	3.071782	-3.453230	-12.953122	37.161745	-4.761860
3.40	0.1000	-0.1000	1.00	1.864249	2.595396	-3.143865	-13.138314	38.680617	-5.237977
3.50	0.1000	-0.1000	1.00	5.732107	2.071098	-3.957295	-12.951890	39.121543	-5.692719
3.60	0.1000	-0.1000	1.00	9.444461	1.522427	-4.079866	-12.318559	38.476619	-5.520268
3.70	0.1000	-0.1000	1.00	13.492123	0.970455	-4.110974	-11.414213	36.774770	-5.326430
3.80	0.1000	-0.1000	1.00	17.169480	0.447757	-4.051994	-10.085000	34.080023	-4.520347
3.90	0.1000	-0.1000	1.00	20.577602	0.084280	-3.903898	-8.348040	30.488666	-4.319123
4.00	0.1000	-0.1000	1.00	23.626469	-0.486193	-3.675083	-6.250280	26.125358	-3.545526
4.10	0.1000	-0.1000	1.00	26.239005	-0.840785	-3.374106	-3.927633	21.138435	-2.629211
4.20	0.1000	-0.1000	1.00	28.352042	-1.103713	-3.011847	-1.303733	15.694485	-1.001522
4.30	0.1000	-0.1000	1.00	29.922193	-1.283985	-2.601182	1.531352	9.972447	-0.499646
4.40	0.1000	-0.1000	1.00	30.919141	-1.313870	-2.156337	4.523782	4.157415	0.649538
4.50	0.1000	-0.1000	1.00	31.231123	-1.245876	-1.692282	7.615736	-1.363640	1.778700
4.60	0.1000	-0.1000	1.00	31.140120	-1.107300	-1.226720	10.745264	7.018007	2.879218

6.70	0.110	1.10	30.46012	-0.764821	-0.768567	17.867124	-17.021500	7.504124
6.80	0.110	1.10	30.77312	-0.797865	-0.799825	16.914077	16.461114	7.504124
6.90	0.110	1.10	31.117462	0.085755	0.082727	19.892481	0.216130	7.504124
7.00	0.110	1.10	31.409121	0.656359	0.591702	17.414571	98.0705	7.504124
7.10	0.110	1.10	31.804310	1.324336	0.670065	15.214571	98.1107	7.504124
7.20	0.110	1.10	31.885221	2.928390	0.838576	17.266400	10.1944	7.504124
7.30	0.110	1.10	31.805600	0.396687	0.611637	18.468577	34.742721	7.504124
7.40	0.110	1.10	31.788731	5.479316	0.572111	18.512571	12.043170	7.504124
7.50	0.110	1.10	31.947339	0.107112	0.172675	27.748744	-10.146074	7.504124
7.60	0.110	1.10	31.761946	6.020090	0.321487	10.434483	14.885445	7.504124
7.70	0.110	1.10	31.370371	6.171919	0.547621	24.977246	46.616600	7.504124
7.80	0.110	1.10	31.617131	8.712203	-1.339087	22.609860	76.247219	7.504124
7.90	0.110	1.10	31.067247	4.776974	-1.723600	12.768143	101.143076	7.504124
8.00	0.110	1.10	31.201482	3.509440	-1.976156	22.608898	114.565252	7.504124
8.10	0.110	1.10	31.278199	2.697812	-2.009574	23.621684	110.714769	7.504124
8.20	0.110	1.10	31.090278	0.732040	-1.812180	25.698924	92.0141350	7.504124
8.30	0.110	1.10	31.054117	-0.395131	-1.392032	28.803932	59.706244	7.504124
8.40	0.110	1.10	31.035057	-1.121500	-0.785027	32.990353	22.776891	7.504124
8.50	0.110	1.10	31.212684	-1.411132	0.083209	37.693262	-3.864960	7.504124
8.60	0.110	1.10	31.226888	-1.200415	0.736587	42.665160	-37.446639	7.504124
8.70	0.110	1.10	31.381124	-0.681234	1.517261	47.577805	-67.223743	7.504124
8.80	0.110	1.10	31.496445	0.304485	2.223361	52.115591	-76.305041	7.504124
8.90	0.110	1.10	31.828994	0.050089	2.792913	55.961936	-65.141171	7.504124
9.00	0.110	1.10	31.241487	0.905412	3.243352	59.084650	-60.318480	7.504124
9.10	0.110	1.10	31.482933	0.319981	3.670423	61.536312	-62.544644	7.504124
9.20	0.110	1.10	31.220777	0.009841	4.112416	63.384610	-64.815755	7.504124
9.30	0.110	1.10	31.471101	-0.603342	4.352574	64.607468	-66.929091	7.504124
9.40	0.110	1.10	31.961508	-0.607106	4.974303	65.162167	-68.695122	7.504124
9.50	0.110	1.10	31.961508	-0.607106	4.974303	65.162167	-68.695122	7.504124

[illegible]

TABLE 1COMPARISON OF STABILITY AND CONTROL DERIVATIVES

Derivative	Value obtained by modified N.R.method	Value obtained by D.F.P.method	Value obtained by Ref.17
$L_p$	-0.2575	-0.5088	-0.2740
$L_r$	-0.1513	0.2078	0.0628
$L_\beta$	-13.4469	-11.9081	-11.4000
$N_p$	+0.0015	0.0380	0.0013
$N_r$	-0.1760	-0.2242	-0.2081
$N_\beta$	2.4986	2.4513	2.1701
$Y_p$	0.0773	0.0702	0.0083
$Y_\beta$	-0.6788	-0.3709	-0.3150
$L_{\delta_a}$	8.0890	9.3776	8.2232
$L_{\delta_r}$	3.4846	4.9674	4.1772
$N_{\delta_a}$	-1.1231	-1.0185	-1.1406
$N_{\delta_r}$	-3.5475	-3.5062	-3.7614

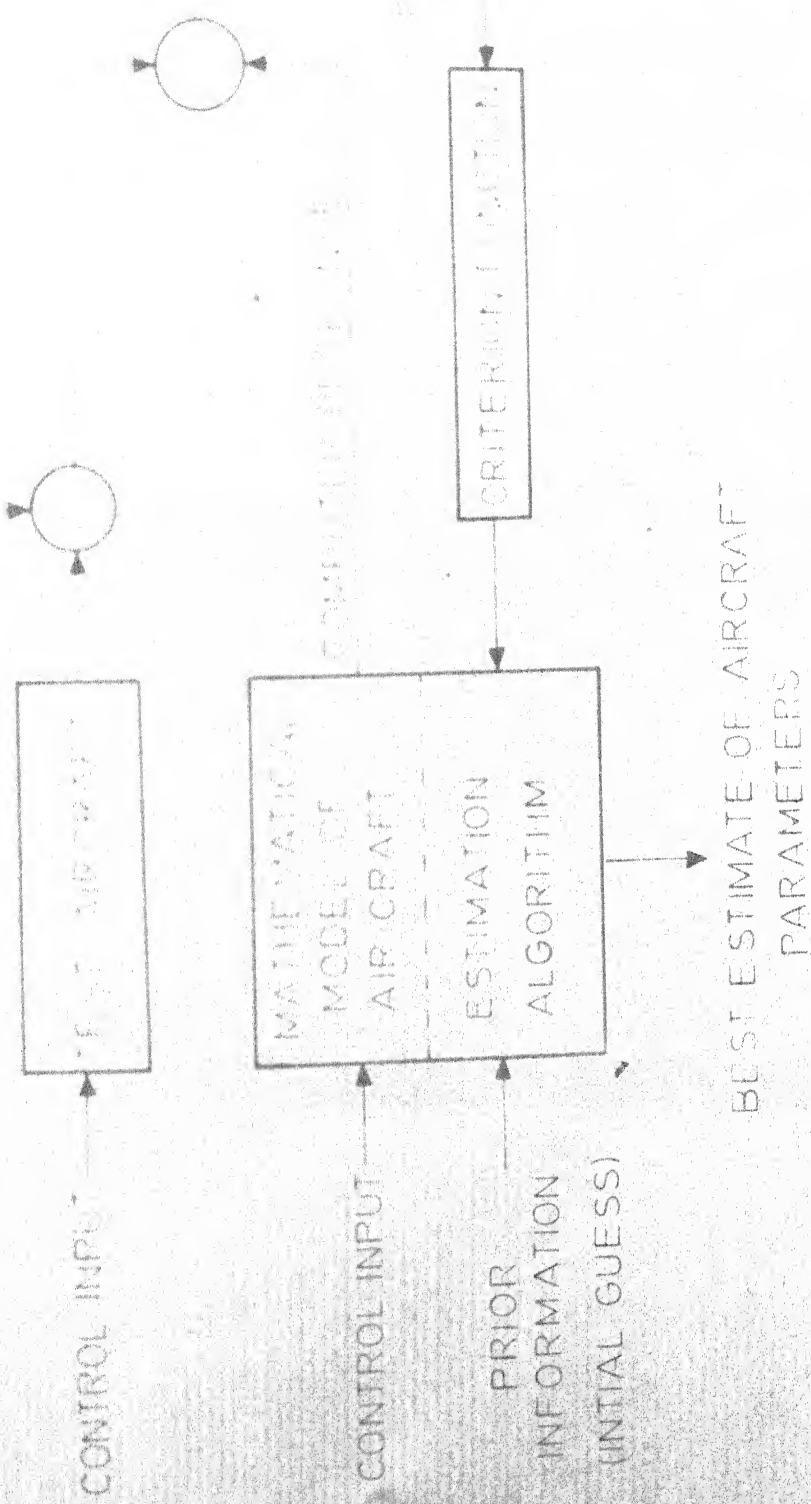


FIG.1 BASIC CONCEPT OF AIRCRAFT PARAMETER ESTIMATION TECHNIQUE

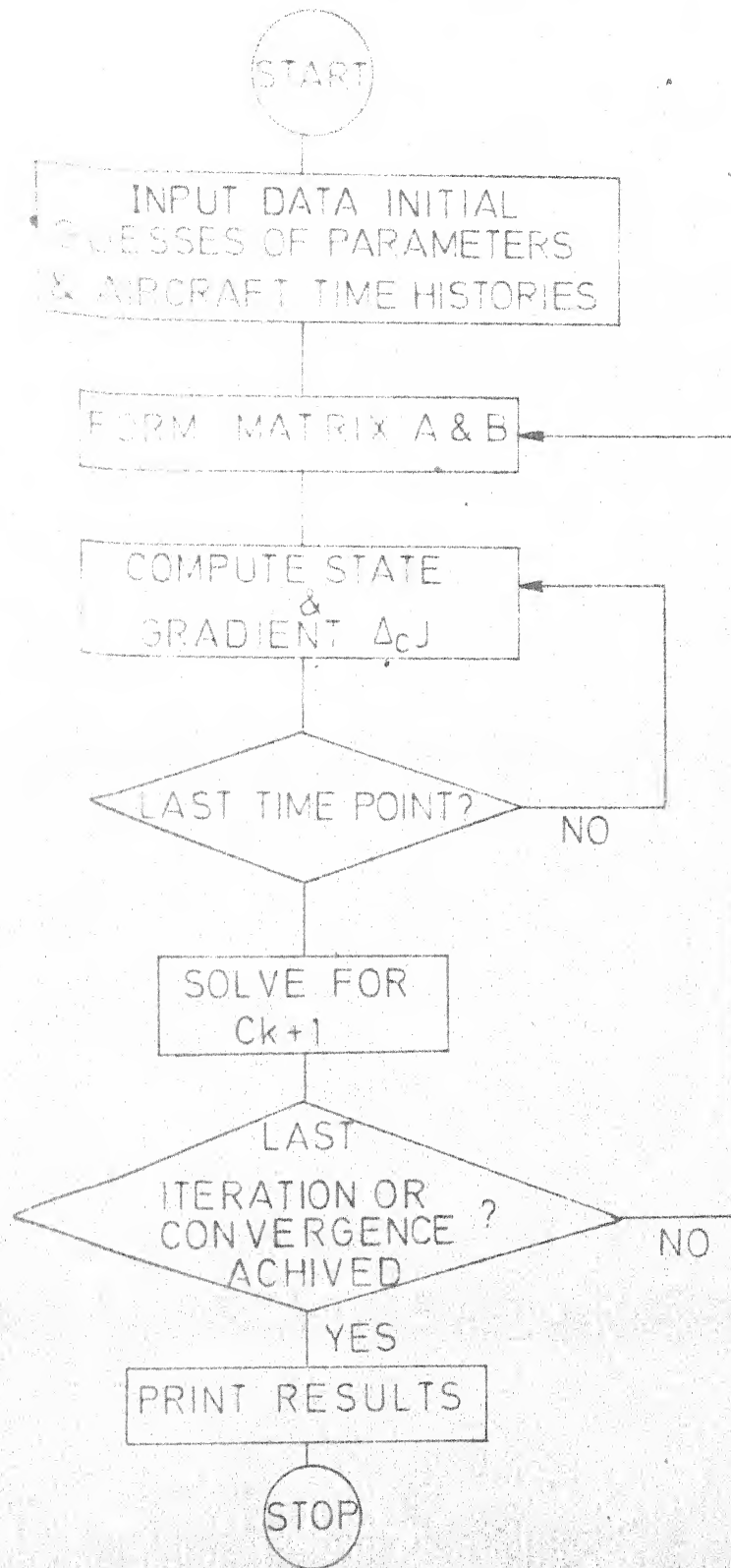


FIG.2 LOGICAL FLOW DIAGRAM FOR COMPUTER PROGRAM OF MODIFIED NEWTON RAPHSON TECHNIQUE & DAVIDON-FLETCHER-POWELL TECHNIQUE

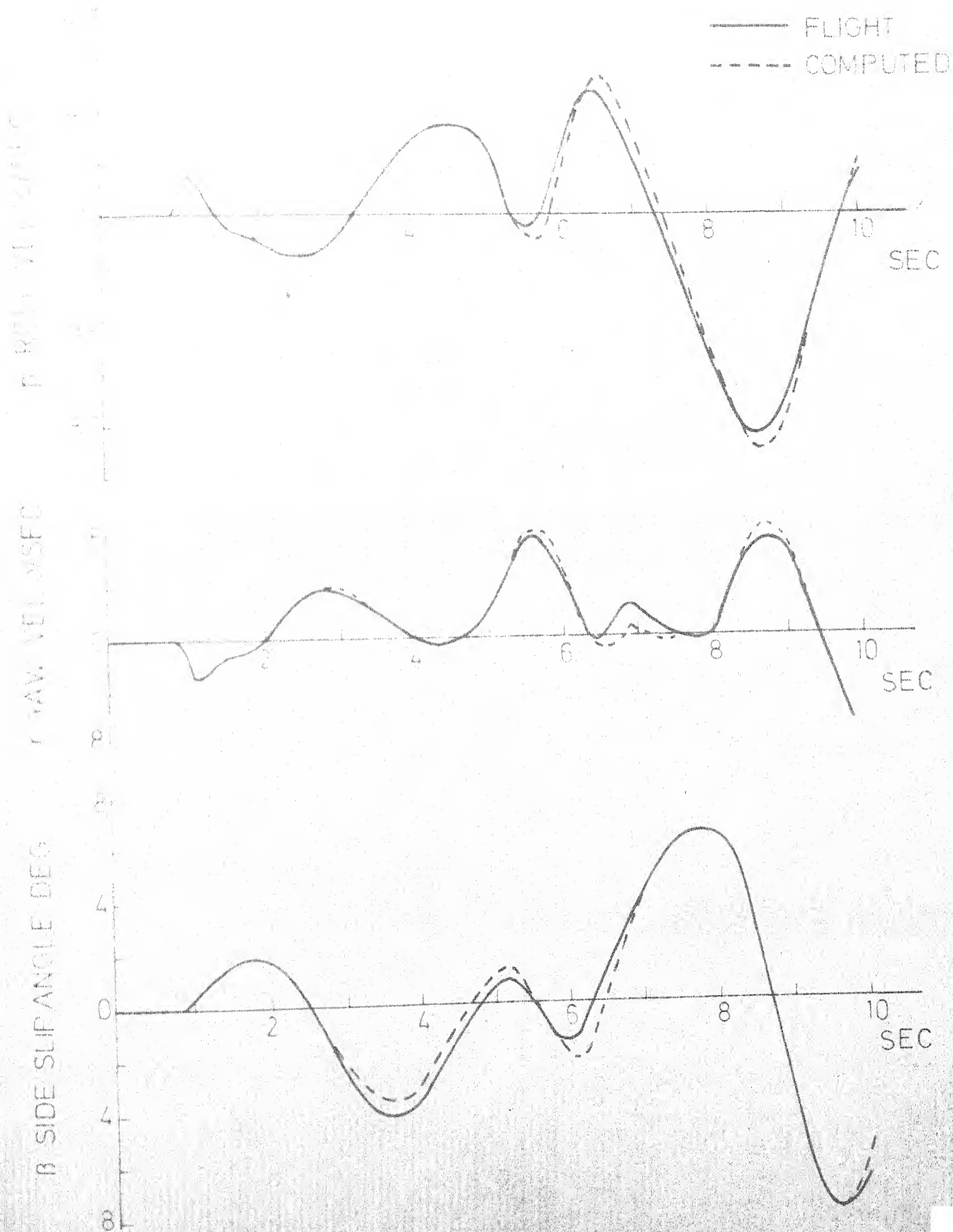


FIG 3 COMPARISON OF TIME HISTORIES MEASURED IN FLIGHT  
& COMPUTED BY MODIFIED NEWTON RAPHSON METHOD

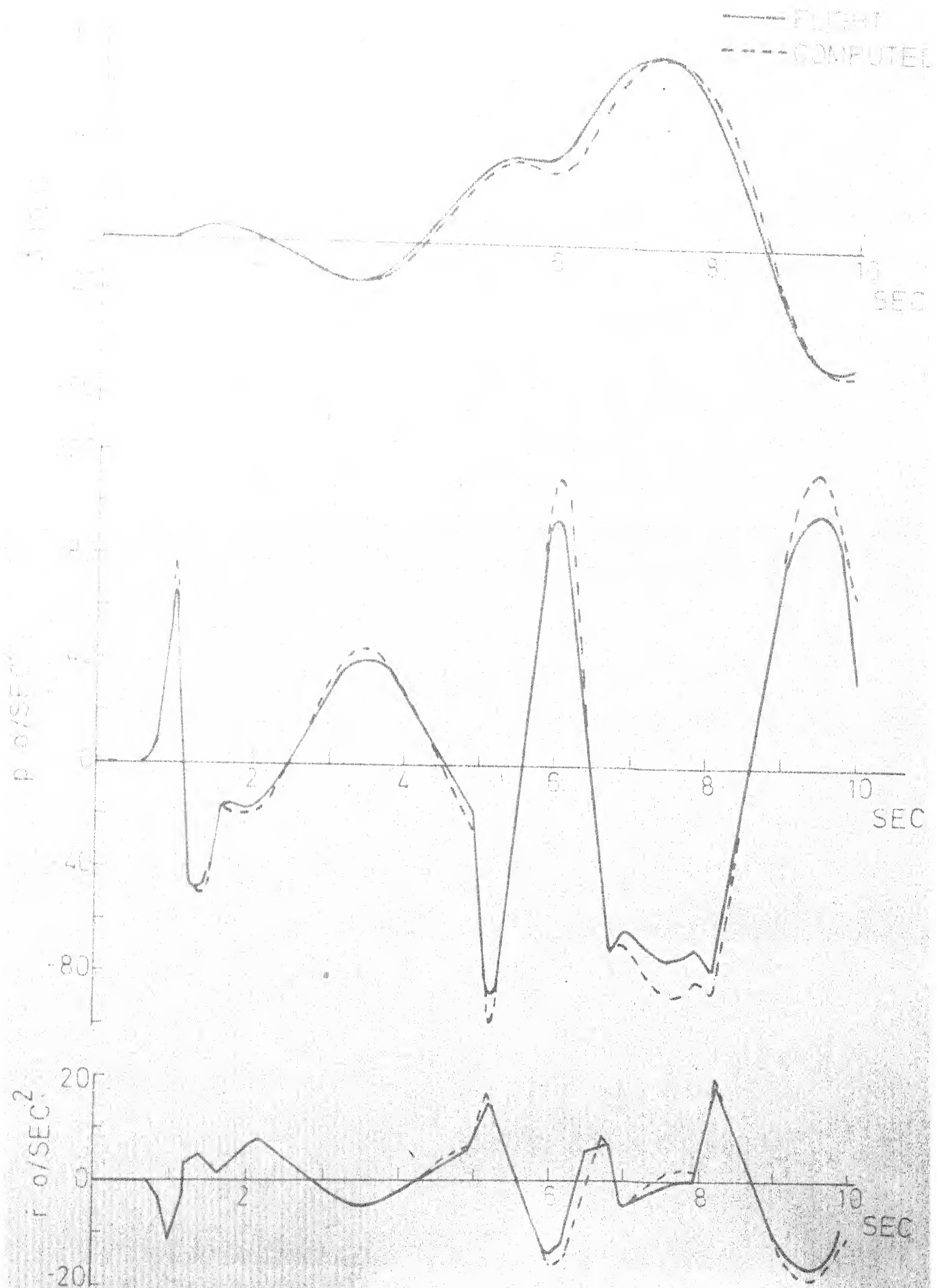


FIG. 2. (CONT.)

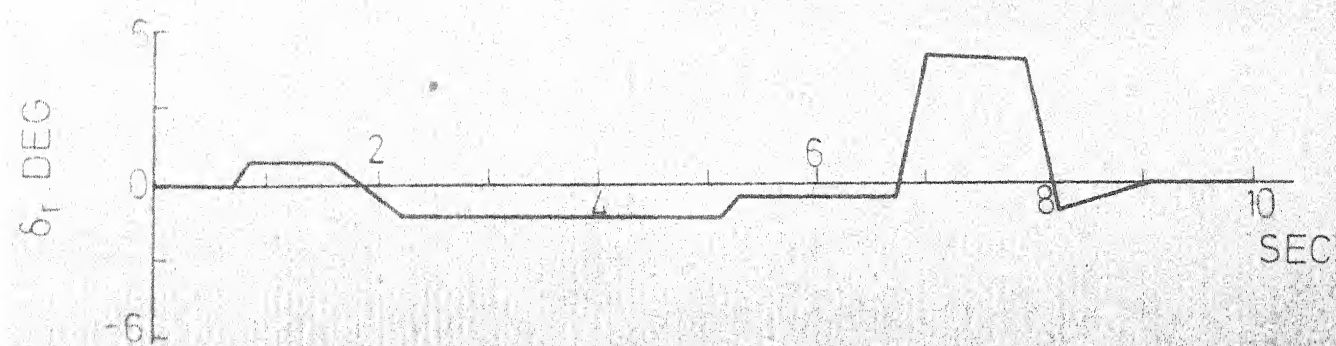
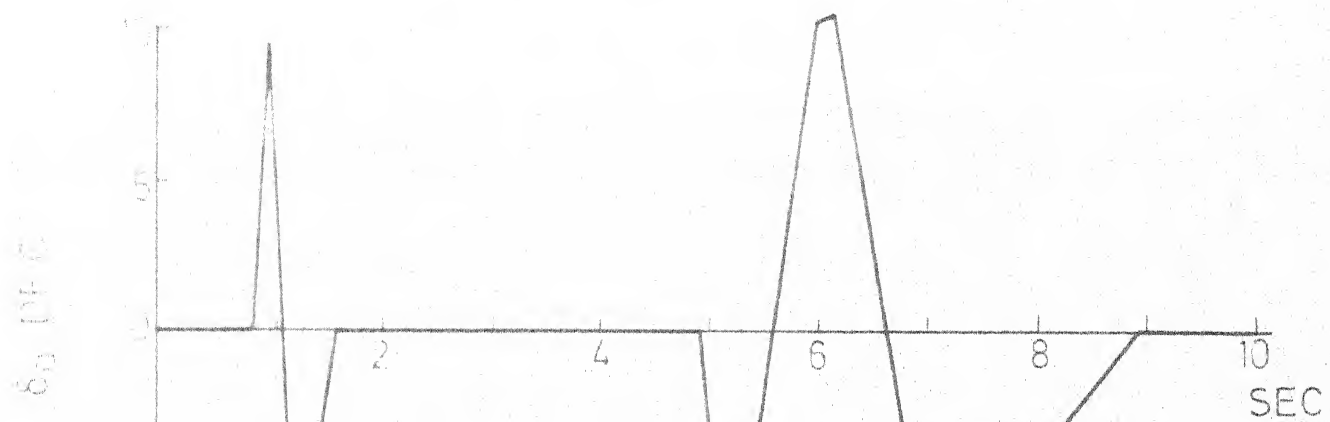
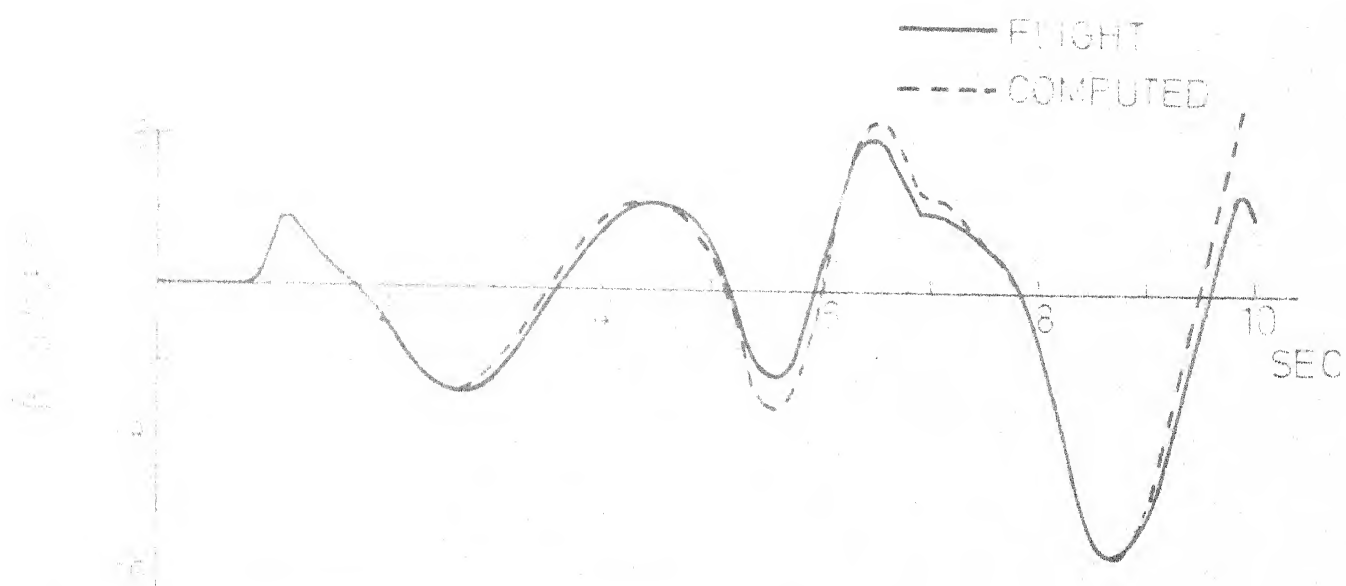


FIG. 3 (CONT)

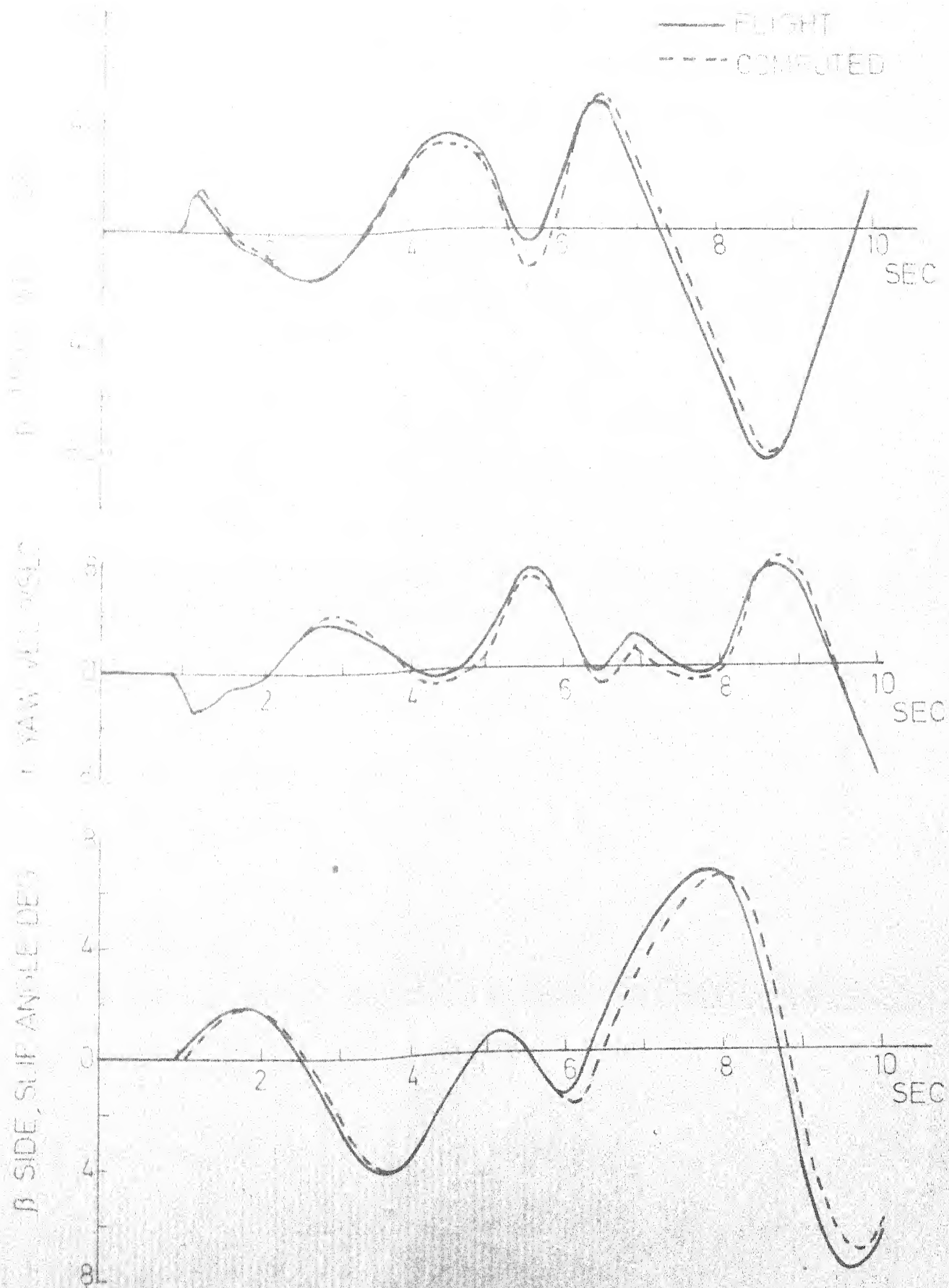


FIG-4 COMPARISON OF TIME HISTORIES MEASURED IN FLIGHT & COMPUTED BY D.F.P. METHOD

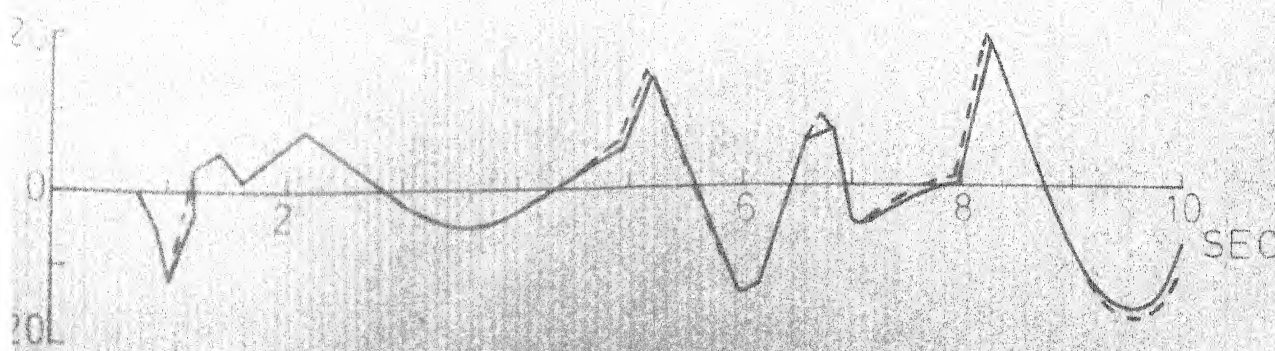
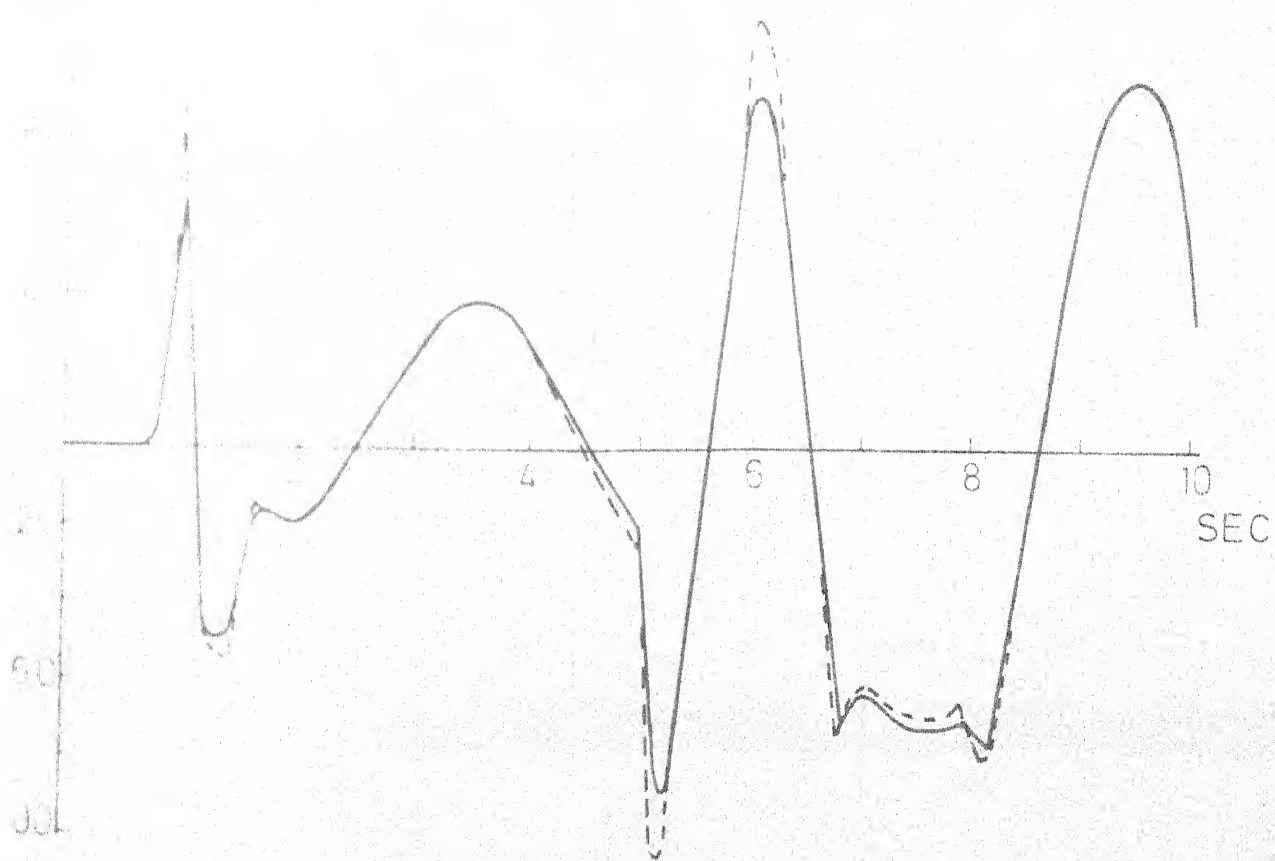
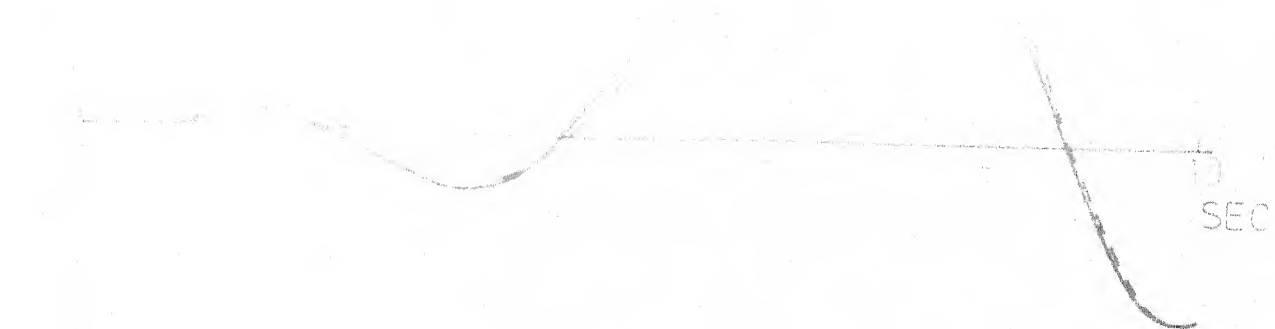


FIG 4 (CONT)

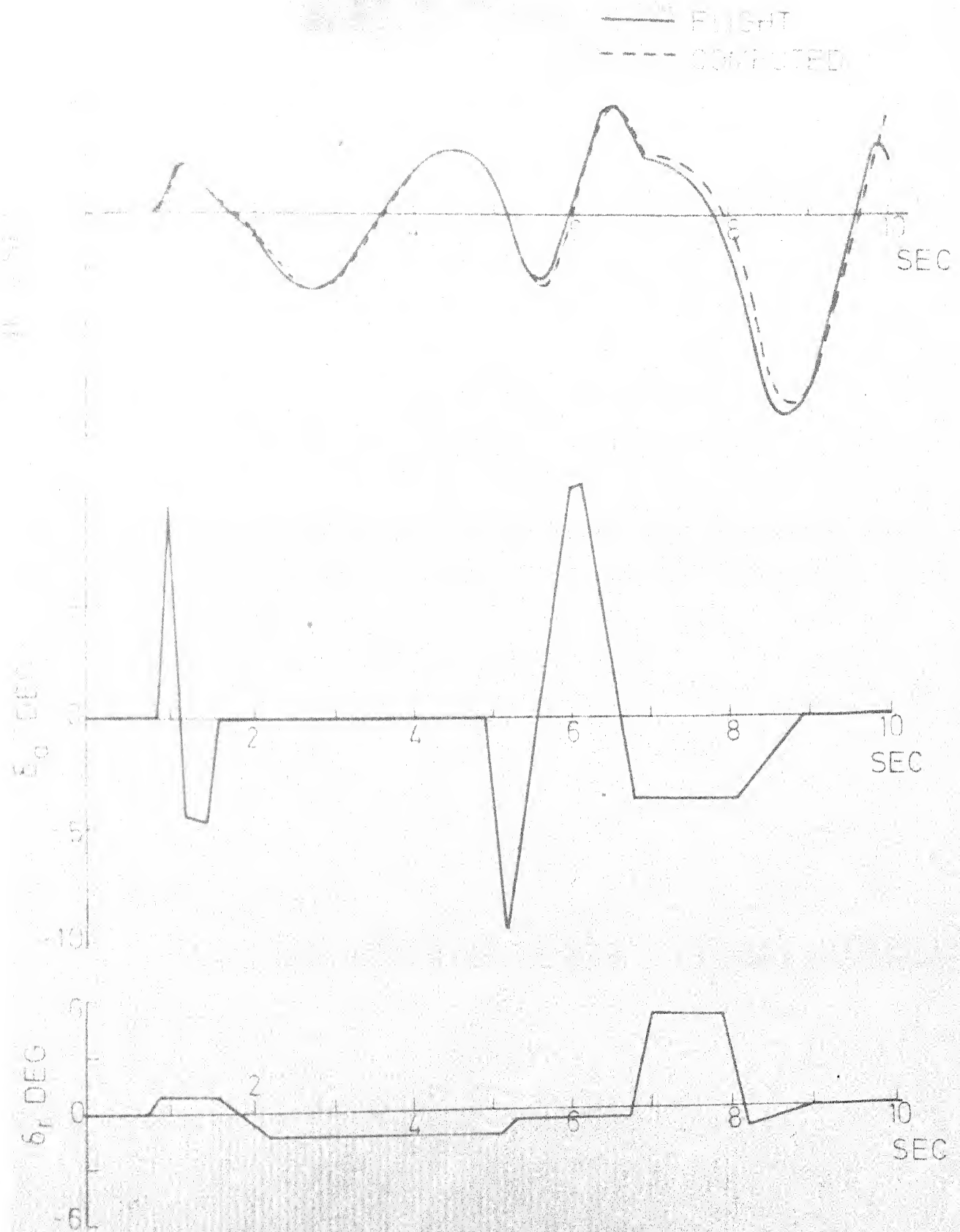


FIG 4 (CONT)